# CALCULATION AND REPRESENTATION GUIDANCE

## Thorndown Primary School

## 2022

This policy outlines the learning journey our children go on surrounding calculation at Thorndown Primary School. We recognise that 'fluency' is not just about remembering facts. Fluency is underpinned by deep understanding and number sense. Pupils need to develop the ability to use effective strategies to derive facts, prior to being able to recall them quickly. Through their use of maths, pupils need to demonstrate fluency through:

- accuracy
- efficiency
- flexibility

This policy demonstrates how we guide learning through a series of small steps with appropriate scaffolds and challenges to support differing needs. It demonstrates our emphasis on the importance of representation and structure and our use of the concrete, pictorial and abstract approach. Our mathematical vocabulary and stem sentences which help children learn new concepts and give them the scaffold on which to verbalise their learning are also highlighted. Teachers will follow this guide to calculation and if they need to deviate they will consult the maths lead.

Below are the contents. You can access the different areas by pressing control and clicking on the year group.

Contents

# Addition and Subtraction

# Addition and Subtraction

## Year 1

## Compose and Partition Numbers to 10 (1)

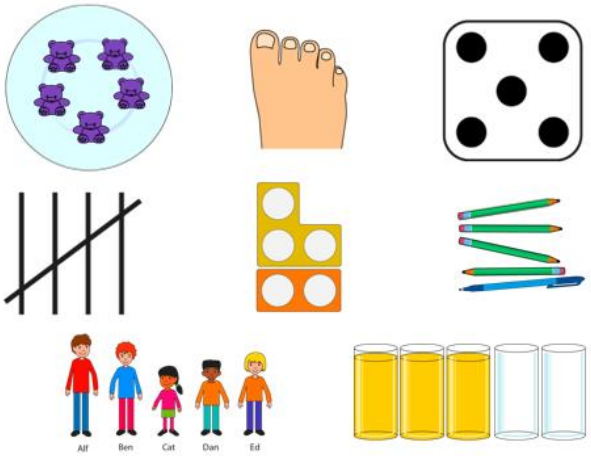
**Vocabulary:**

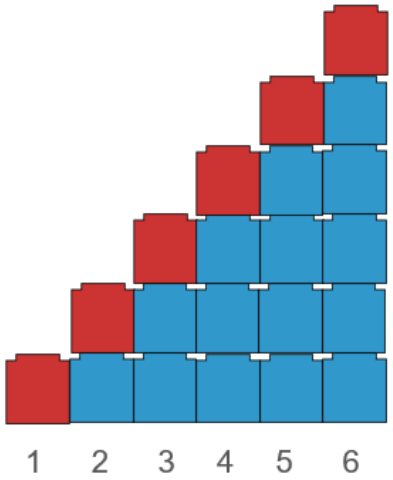
Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Numberblocks Part-Part-Whole model
Ten Frame Fingers Five and-a-bit Systematic Subitise
One more One less




**Understand that numbers to 10 can be represented in many different ways.**

**Numbers to 5 can be identified without counting (subitising).**





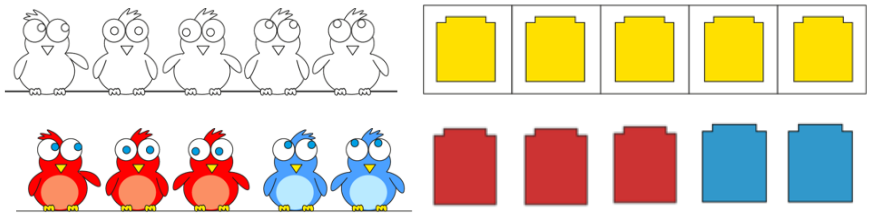

**Each number is composed of the previous number and one more.**

**A number can be partitioned in different ways.**

**There are 5 ___. 3 are ____. 2 are ____.**

*There are 2 glasses. 3 glasses are full and 2 glasses are empty.*
*There are 5 cubes. 3 are red and 2 are blue.*
*There are 5 cakes. 2 have cherries and 3 do not.*

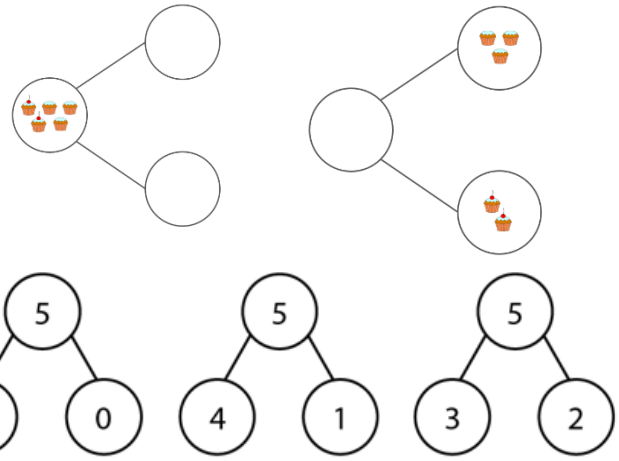
**5 is the whole. 3 is a part. 2 is a part.**



**Each number can be partitioned into two smaller numbers**

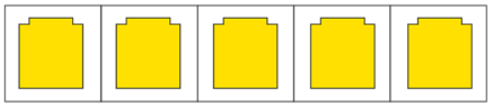
**There are 5 ____. 3 are ____. 2 are ____.**

**5 is the whole. 3 is a part. 2 is a part.**




Remember to show PW models in different orientations.

Do not refer to zero as a 'part' of another number.



## Addition and Subtraction

## Year 1

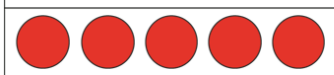
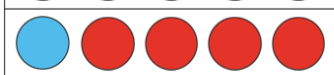
## Compose and Partition Numbers to 10 (2)

**Vocabulary:**

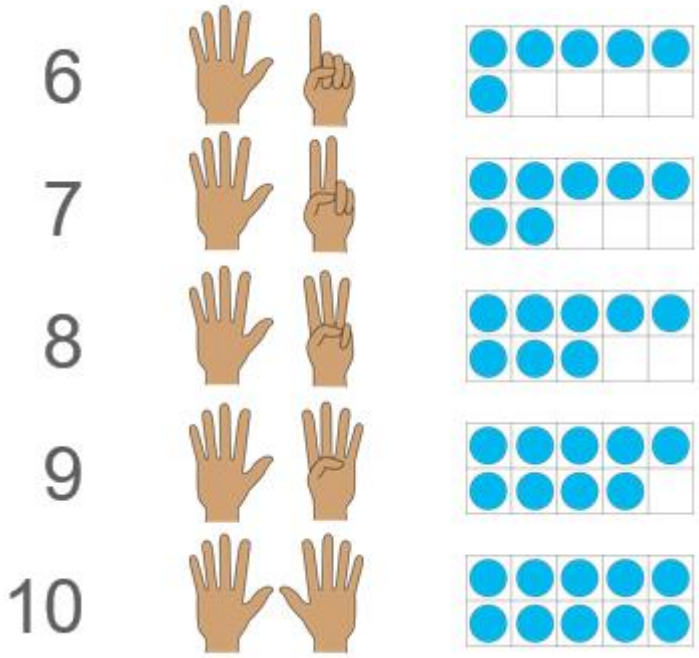
Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Numberblocks Part-Part-Whole model
Ten Frame Fingers Five and-a-bit Systematic Subitise
One more One less

There are two types of subitising

1. Perceptual – Just 'seeing' the group and knowing how many (up to 5 in non-standard arrangements). Supports cardinality.
2. Conceptual – Seeing groups within groups. Supports composition and enables us to subitise larger amounts.

| | Blue | Red |
|---|---|---|
|  | 0 | 5 |
|  | 1 | 4 |
|  | 2 | 3 |
|  | 3 | 2 |
|  | 4 | 1 |
|  | 5 | 0 |

**A number can be partitioned in different ways systematically.**




**Numbers from 6 – 10 are composed of the '5 and a bit' structure.**

# Addition and Subtraction

## Year 1

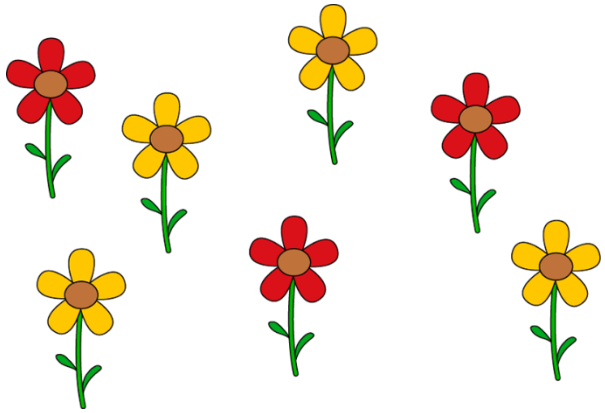
## Read, Write and Interpret Additive Equations (1)

**Vocabulary:**
Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Total Part-Part-Whole model Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equals = Is equal to = Addition Subtraction Quantity Increase Decrease First, Then, Now Expression Equation

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

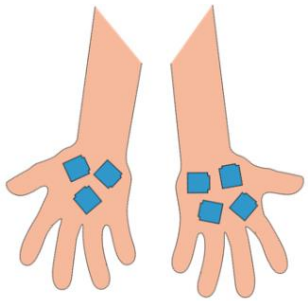


**Identify what each number represents using real life contexts.**

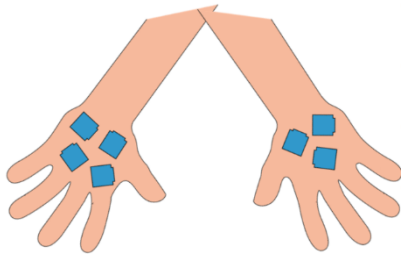
**The 4 represents the 4 yellow flowers.**

**The 3 represents the 3 red flowers.**

Start with lots of talk, before introducing the abstract symbols/equations alongside the actions/ images.



3 + 4

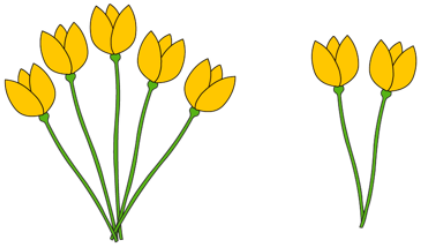


4 + 3

Note the progression in the use of resources/ images, starting with identical objects, where the groups are distinguished by position or colour.

**We can write the addends in any order.**
**(Commutative Law)**
*3 plus 4 is equal to 3 plus 4*

An expression does not have '=' : 3 + 4
An equation includes '=' : 3 + 4 = 7, 7 = 4 + 3

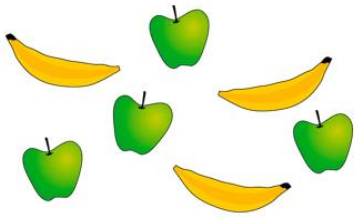


5 + 2 = 7

**Identify what each number represents in an equation.**

**We can write 5 plus 2 is equal to 7.**

**The 5 represents ____.**
**The 2 represents___.**
**The 7 represents the total number of ___.**
**The 7 represents how many ___there are in all.**



4 + 3 = 7
7 = 4 + 3

**USE the NOUNS**
5 flowers plus 2 flowers is equal to 7 flowers.
4 apples plus 3 bananas is equal to 7 pieces of fruit.

**Vary the position of the = symbol**

The first addend in an expression or equation can also be called the augend.

# Addition and Subtraction

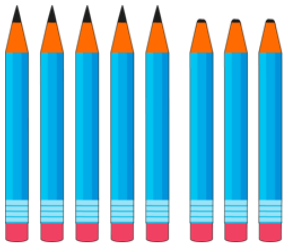
## Year 1

## Read, Write and Interpret Additive Equations (2)

**Vocabulary:**
Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Total Part-Part-Whole model Frame Fingers Five and-a-bit Systematic Plus + Minus - Equals = Is equal to = Addition Subtraction Quantity Increase Decrease First, Then, Now Expression Equation

**Addend + Addend = Sum (or Augend + Addend = Sum)**

**Minuend – Subtrahend = Difference**



8 – 5 = 3

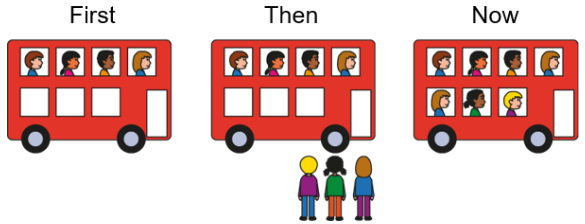
**Subtraction can take the form of partitioning.**

**There are 8 ____ altogether.**

**5 ___ are ____.**

**3 ___ are ____.**

**We can write this as 8 minus 5 is equal to 3.**




**4 + 3 = 7**

**Addition can tell us about combining objects. (Aggregation structure)**

**Subtraction can tell us about splitting objects into two or more groups. (Partitioning structure)**

The partitioning structure is sometimes referred to as the 'not structure'. Eg, There are 5 teddies. 3 are in the tent and 2 are not.

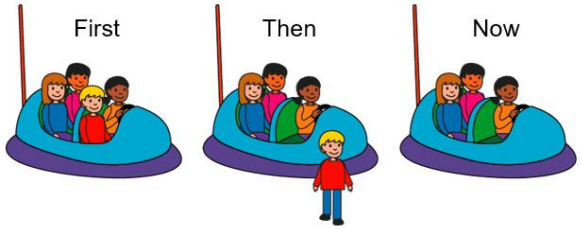
**Make connections between addition and subtraction.**

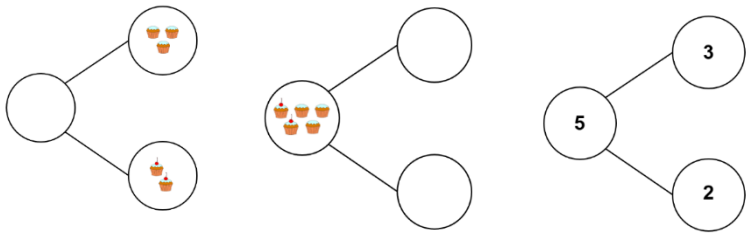
**This can be shown using the part-part-whole model. Ensure children have lots of practice in combining and partitioning objects before using abstract numerals.**

**Understand the 'First, Then, Now' structure of addition and subtraction.**

**Addition can tell us about a quantity increasing. (Augmentation)**

**Subtraction can tell us about a quantity decreasing. (Reduction)**



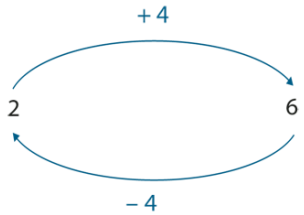

2 + 3 = 5

3 + 2 = 5

5 – 3 = 2

5 – 2 = 3

**4 – 1 = 3**




**Addition and subtraction undo each other.**

## Addition and Subtraction

## Year 2

## Add and Subtract across 10 (1)

**Vocabulary:**

Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Total Part-Part-Whole model Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equals = Is equal to = Addition Subtraction Quantity Increase Decrease First, Then, Now Expression Equation

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

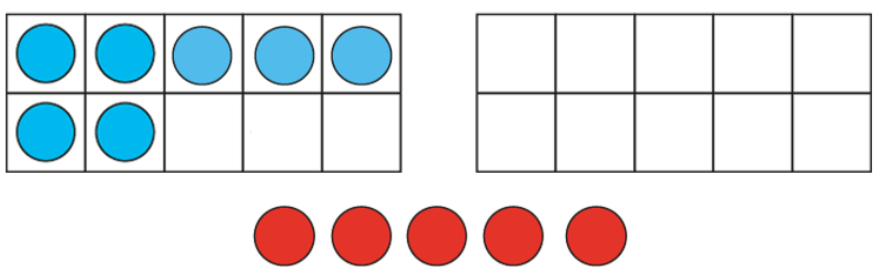
***We subtract the subtrahend.***

**Use knowledge of known facts to bridge through 10 using a 'make 10' strategy. We can partition one of the addends to help us add.**

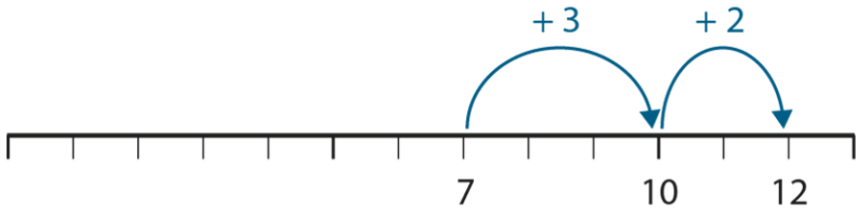
**First, I partition the __ into ___ and ___.**

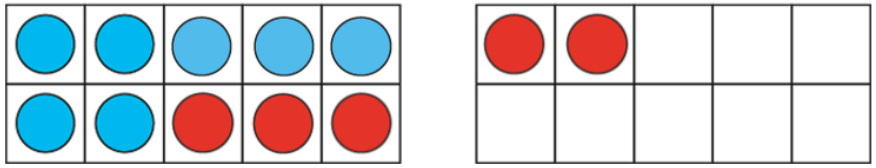
**Then, I add ___ and ___ to make 10.**

**Then, I add the remaining ___ to make ___.**

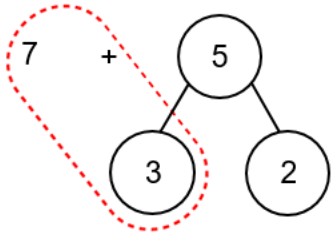


7 + 5



7 + 5 = 7 + 3 + 2 = 10 + 2




7 + 3 = 10

10 + 2 = 12

## Addition and Subtraction

## Year 2

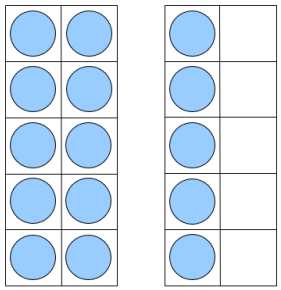
## Add and Subtract across 10 (2)

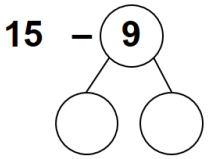
**Vocabulary:**

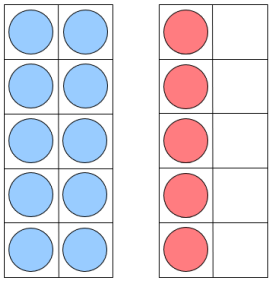
Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Total Part-Part-Whole model Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equals = Is equal to = Addition Subtraction Quantity Increase Decrease First, Then, Now Expression Equation

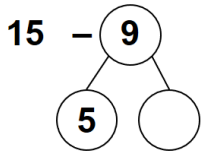
Addend + Addend = Sum Minuend – Subtrahend = Difference

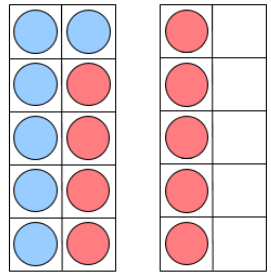
We subtract the subtrahend.

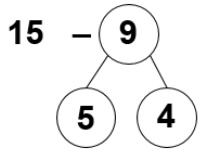








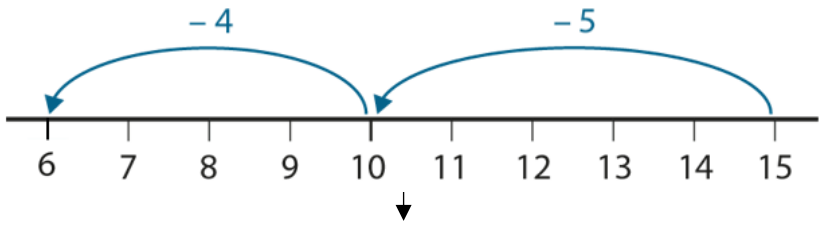

**15 – 9 = 6**

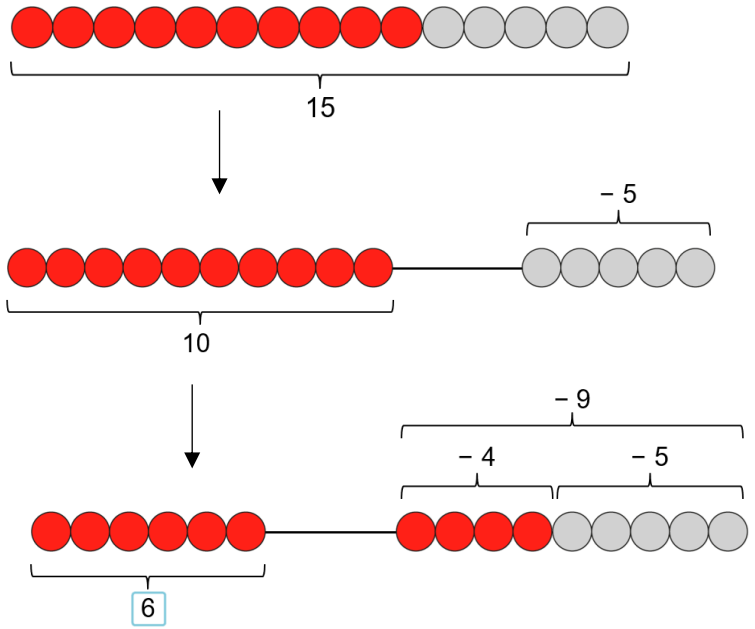
**Use knowledge of known facts to subtract *through 10*. We can partition the subtrahend to help us subtract.**

**First, I partition the __ into ___ and ___.**

**Then, I subtract ___ to get to 10.**

**Then, I subtract the remaining ___ to make ___.**



## Addition and Subtraction

## Year 2

## Add and Subtract across 10 (3)

**Vocabulary:**

Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Total Part-Part-Whole model Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equals = Is equal to = Addition Subtraction Quantity Increase Decrease First, Then, Now Expression Equation

Addend + Addend = Sum

Minuend – Subtrahend = Difference

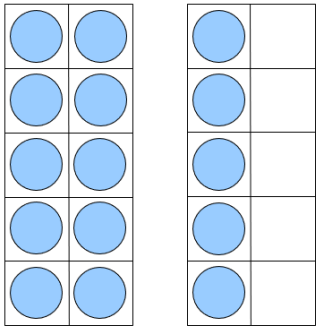
**Use knowledge of known facts to subtract *from 10*. We can partition the subtrahend to help us subtract.**

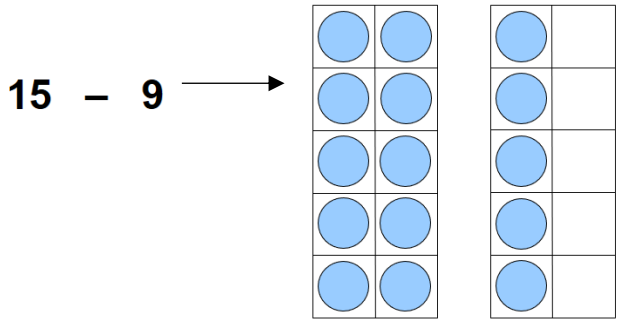
**First, I partition the __ into ___ and ___.**

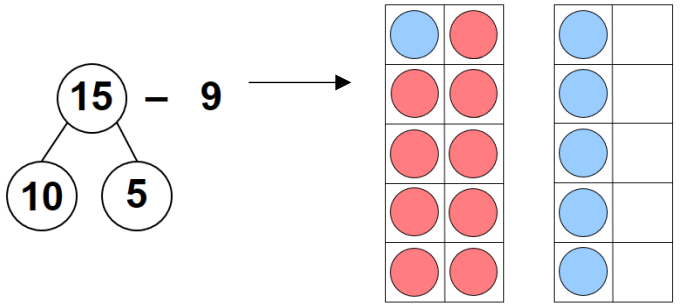
**Then, I subtract ___ from 10 to make ___.**

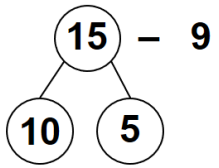
**Then, I add the remaining ___ to make ___.**

We subtract the subtrahend.






**10 – 9 = 1**

**1 + 5 = 6**

**15 – 9 = 6**

The rekenrek is a great tool for modelling this strategy.

Show the number using 10 beads on the top row and 5 on the bottom.

Subtract 9 from the top row by moving them out of play (across to the right).

This leaves 1 bead on the top row that can be combined with the 5 on the bottom row.

# Addition and Subtraction

## Year 2

## Solve Comparative Addition and Difference Problems

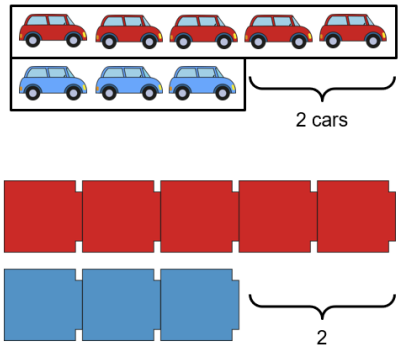
**Vocabulary:**

Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Total Part-Part-Whole model Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equals = Is equal to = Addition Subtraction Quantity Increase Decrease First, Then, Now Expression Equation Difference Bar model

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

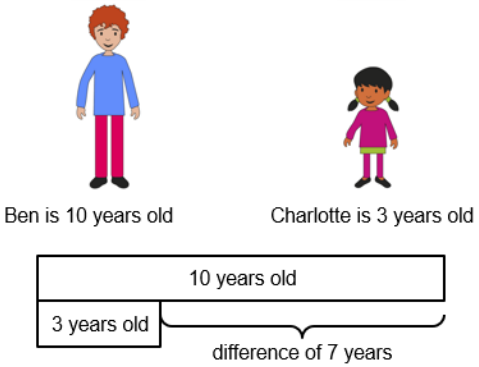



**Line up sets of objects in a bar model structure to support comparison.**

**There are 2 fewer blue cars than red cars.**

**There are 2 more red cars that blue cars.**

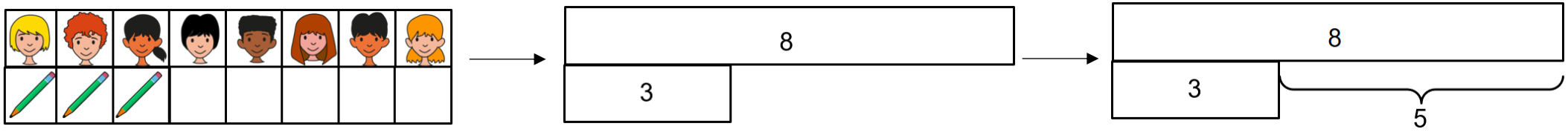
**The difference is 2 cars.**




**Represent a range of comparison contexts.**

**Ben is 7 years older than Charlotte.**

**Charlotte is 7 years younger than Ben.**



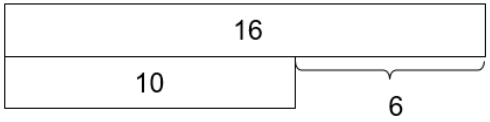

**We can use subtraction to help solve difference problems / missing addend problems about 'how many more?' and 'how many fewer?'**

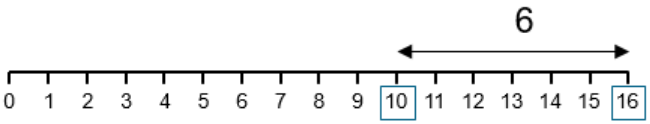
**3 + __ = 8**

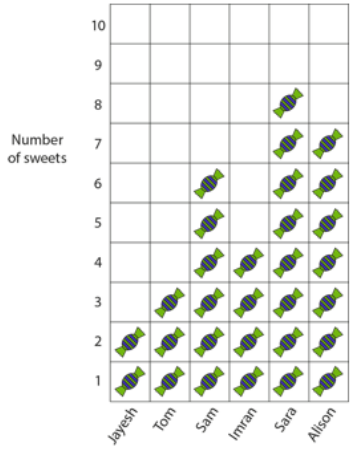
**8 – 3 = 5**

**Create contexts for recognising the difference/comparative addition structure with all representations below.**

**10 + ☐ = 16** **16 – 10 = ☐**



## Addition and Subtraction

## Year 2

## Add and Subtract within 100 (1)

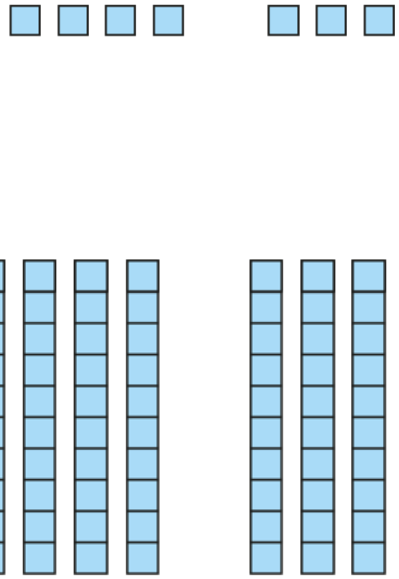
**Vocabulary:**

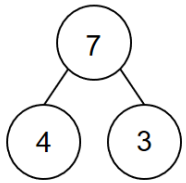
Part Whole Ones Tens Represents Compose Combine Partition Total Part-Part-Whole model Tens Frame Dienes Plus + Minus - Equals = Is equal to = Addition Subtraction Expression Equation Regroup Number line Tens Boundary

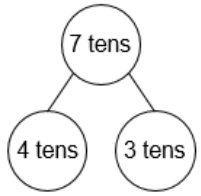
**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***





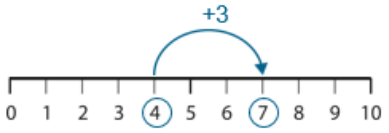

**Use known facts within 10 to add/subtract multiples of 10.**

**I know that 4 plus 3 is equal to 7.**

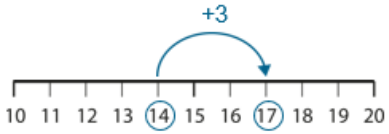
**So, 4 tens plus 3 tens is equal to 7 tens.**

**40 + 30 = 70.**

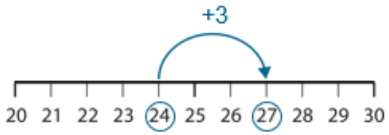
**70 – 40 = 30**



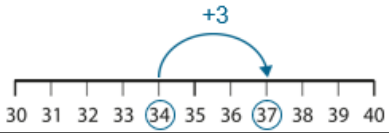

4 + 3 = 7



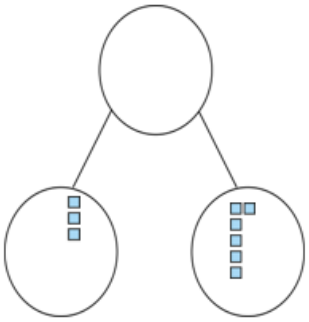

14 + 3 = 17



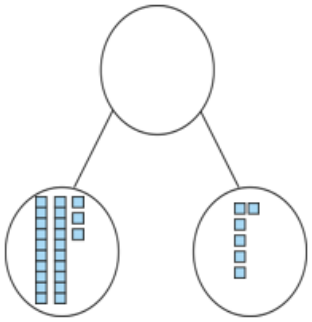

24 + 3 = 27




34 + 3 = 37



3 + 6 = 9



23 + 6 = 29

**Use known facts within 10 to add/subtract ones to/from a 2 digit number.**

**I know that 3 plus 6 is equal to 9.**

**So, 2 tens and 3 ones plus 6 ones is equal to 2 tens and 9 ones.**

**23 + 6 = 29.**

**Generalise that adding/subtracting within 10 can be applied to adding a 2 digit number with a 1 digit number – not crossing the tens boundary.**

**I know that 4 plus 3 is equal to 7.**

**So, 1 ten and 4 ones plus 3 ones is equal to 1 ten and 7 ones.**

**4 + 3 = 7**

**So 14 + 3 = 17.**

## Addition and Subtraction

## Year 2

## Add and Subtract within 100 (2)

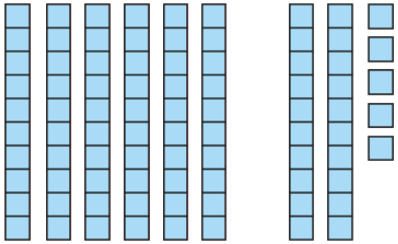
**Vocabulary:**
Part Whole Ones Tens Represents Compose Combine Partition Total Part-Part-Whole model Tens Frame Dienes Plus + Minus - Equal to = Addition Subtraction Expression Equation Regroup Count on Count back Number line Tens Boundary

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**
***We subtract the subtrahend.***

**6 + 2 = 8**

**60 + 25 = ?**



**Use known facts within 10 to add/subtract multiples of 10 to a 2-digit number.**

**I know that 6 plus 2 is equal to 8.**

**So, 6 tens plus 2 tens is equal to 8 tens. Then add the additional 5 ones.**

**60 + 20 = 80.**

**80 + 5 = 85**

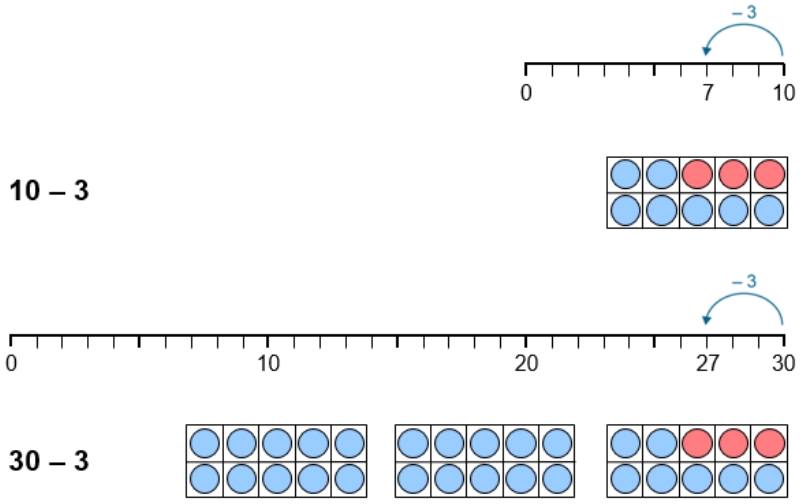
**Or 60 + 25 = 60 + 20 + 5**

**Use knowledge of subtracting from 10 to subtract a single-digit number from a multiple of 10.**

**I know that 10 minus 3 is equal to 7.**

**So, 3 tens minus 3 ones is equal to 2 tens and 7 ones.**

**30 - 3 = 27.**




Provide lots of opportunities for children to subtract a single digit number from a multiple of 10, starting with 1 less.

Display sections of number lines and use procedural variation for practice.

Eg 10-3, 20-3, 30-3

Draw attention to the tens and ones digits.

What changes?
What stays the same?

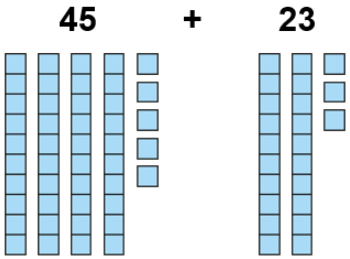
# Addition and Subtraction

## Year 2

## Add and Subtract within 100 (3)

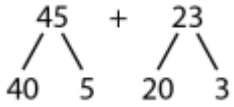
**Vocabulary:**
Part Whole Ones Tens Represents Compose Combine Partition Total
Part-Part-Whole model Tens Frame Dienes Plus + Minus - Equal to = Addition
Subtraction Expression Equation Regroup Number line Tens Boundary
**Addend + Addend = Sum** **Minuend – Subtrahend = Difference**
**Augend + Addend = Sum** ***We subtract the subtrahend.***

## Addition Method A: Partition both addends




**40 + 20 = 60**
**5 + 3 = 8**
**60 + 8 = 68**

**Partition both addends to add efficiently without crossing the tens boundary.**
**(No regrouping)**




Following lots of practice with concrete and pictorial support, move to the use of abstract equations only, with jottings to record the three steps.

First, I partition the 45 into 40 and 5, and the 23 into 20 and 3.

40 + 20 = 60
*'Forty plus twenty is equal to sixty…'*

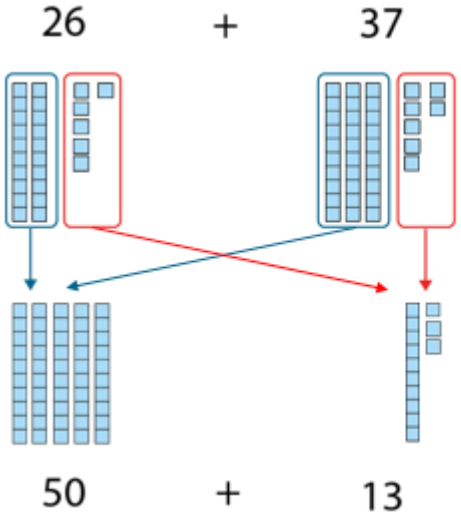
5 + 3 = 8
*'…five plus three is equal to eight…'*

60 + 8 = 68
*'…and sixty plus eight is equal to sixty-eight.'*

45 + 23 = 68

So 45 add 23 is equal to 68.

**Partition both addends to add efficiently when we need to regroup the ones into one ten and some ones.**




20 + 30 = 50
6 + 7 = 13

**Addition and Subtraction**

**Year 2**

**Add and Subtract within 100 (4)**

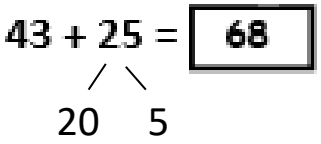
**Vocabulary:**

Part Whole Ones Tens Represents Compose Combine Partition Total
Part-Part-Whole model Tens Frame Dienes Plus + Minus - Equal to = Addition
Subtraction Expression Equation Regroup Number line Tens Boundary

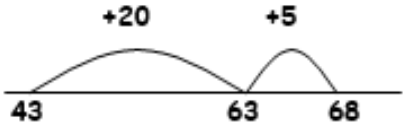
**Addend + Addend = Sum** **Minuend – Subtrahend = Difference**

**Augend + Addend = Sum** ***We subtract the subtrahend.***

## Addition Method B: Partition one addend



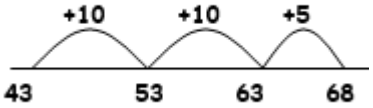

**Partition one addend into tens and ones.**
Add the tens and then the ones.
No need to bridge through a multiple of 10.



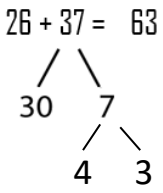

43 = 20 = 63
63 + 5 = 68

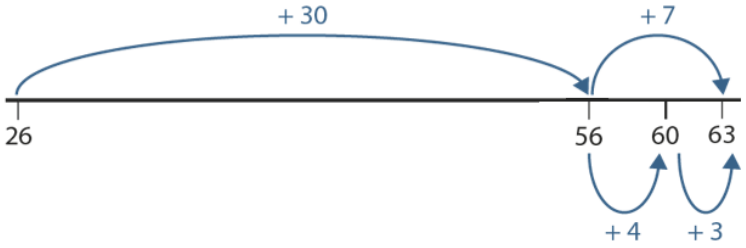
OR 43 + 20 + 5
= 63 + 5
= 68

Interim step if needed




**Partition one addend into tens and ones.**
Add the tens and then the ones.
Bridge through a multiple of 10.




When adding 2-digit numbers, it is really important that children do not use counting strategies (counting on ones or counting manipulatives), but use know facts to add decomposed parts.

If this is the case, provide additional fluency practice, ensuring children know and can apply addition facts to 10 and can add a 1-digit number to a multiple of 10.

To enable successful bridging, children need to be secure in pairs equal to 10 and know why this is important. Intelligent practice in identifying how an addend should be partitioned is very helpful.

Eg I'm adding to 35, how should I partition 6? 7? 8? to bridge through a the next multiple of 10?

I'm adding 8. How should I partition this if I'm adding to 35?, 37?, 32? (It's useful to include examples when partitioning is not necessary. Can they identify when to partition and when not to?)

# Addition and Subtraction

## Year 2

## Add and Subtract within 100 (5)

**Vocabulary:**

Part Whole Ones Tens Represents Compose Combine Partition Total
Part-Part-Whole model Tens Frame Dienes Plus + Minus - Equal to = Addition
Subtraction Expression Equation Regroup Number line Tens Boundary

**Addend + Addend = Sum** **Minuend – Subtrahend = Difference**

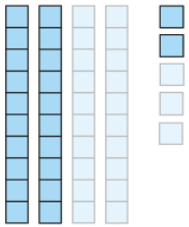
**Augend + Addend = Sum** ***We subtract the subtrahend.***

When subtracting, only partition the subtrahend. (If children learn to partition both the minuend and the subtrahend for calculations when the ones digit in the subtrahend is smaller than the ones digit in the minuend, eg 37-14, they often swap the digits around to try to make it work in calculations such as 34 – 17). This can be a real point of difficulty when children have added by partitioning both addends.

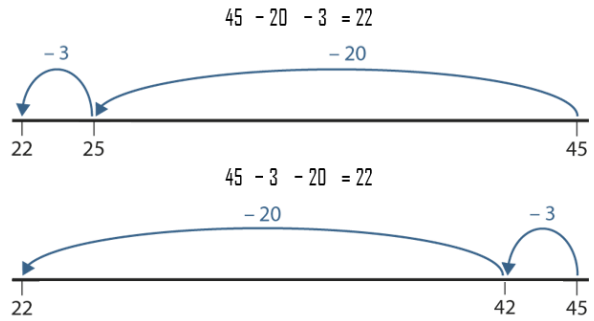
**Subtract from any two-digit number by partitioning the subtrahend into tens and ones.**

**Subtract the tens and then the ones. No bridging through a multiple of 10 initially.**

45 – 20 – 3

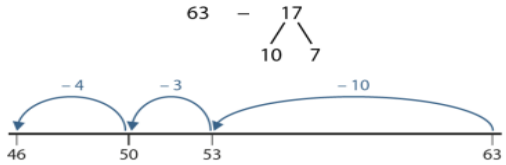


**45 – 23 = 22**

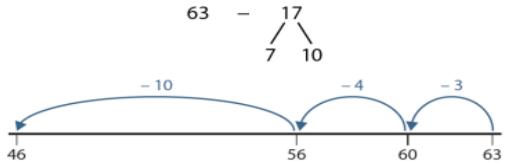



**Subtract two-digit number by partitioning the subtrahend into tens and ones where bridging through a multiple of 10 is required.**

Bridging a multiple of ten – subtracting the tens first:



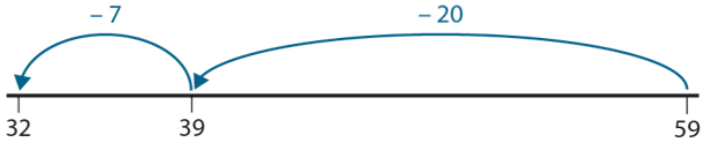

Bridging a multiple of ten – subtracting the ones first:




Provide examples of subtracting the tens first and then the ones first so children understand that this doesn't change the result. Subtracting the tens first can become the preferred strategy, linking in with addition, when adding the tens first is often easier.

59 – 27 = 32

20 7




When subtracting 2-digit numbers, it is really important that children do not use counting strategies (counting on ones or counting manipulatives), but use know facts to add decomposed parts.

If this is the case, provide additional fluency practice, ensuring children know and can apply subtraction facts to 10 and can subtract a 1-digit number from a multiple of 10.

To enable successful bridging, children need to be secure in pairs equal to 10 and know why this is important. See ideas for adding and adapt for subtraction practice.

## Addition and Subtraction

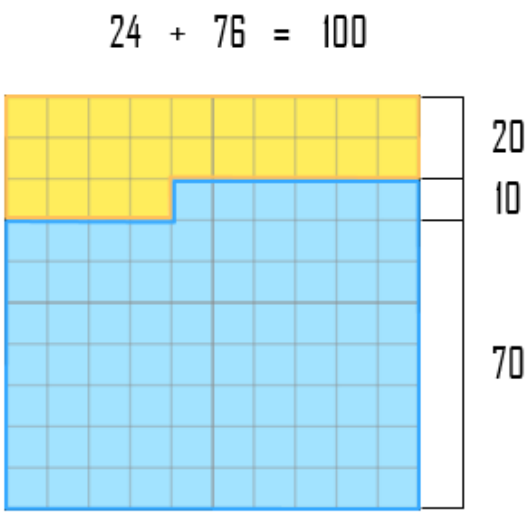
## Year 3

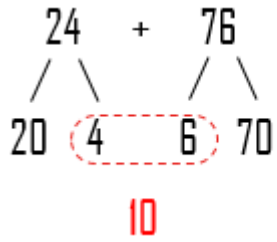
## Calculate complements to 100.

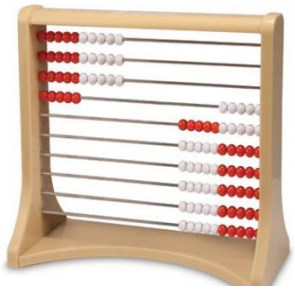
**Vocabulary:**

Part Whole Ones Tens Represents Compose Combine Partition Total Part-Part-Whole model Dienes 100 square Plus + Minus - Is equal to = equals Addition Subtraction Expression Equation Regroup Complements

Addend + Addend = Sum

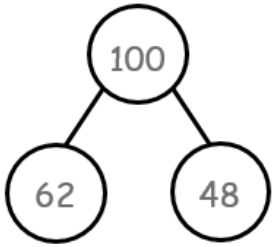


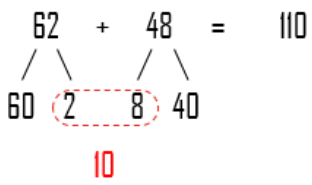



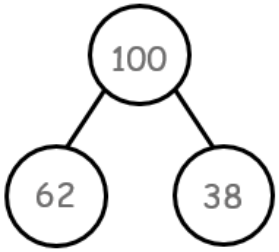



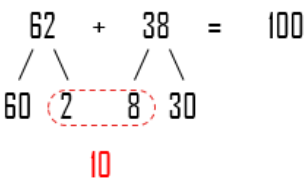
**The 100 bead rekenrek provides a very supportive representation of pairs equalling 100.**

**This is particularly helpful to support children to understand why there are 9 complete tens which combine with the ones, and why 45 + 65 is not 100.**



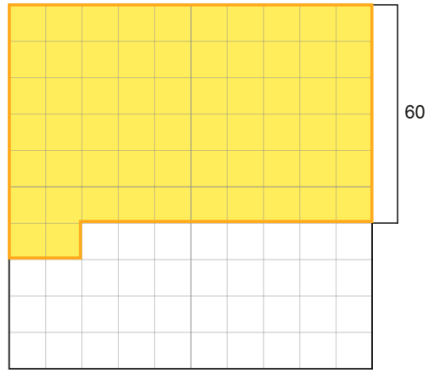




**Use knowledge of subtracting from 10 to subtract a single-digit number from a multiple of 10.**

**First we make 10 ones. The ones digits add up to make 1 ten, so we need 9 more tens to make a total of 100.**

**Compare equations which do and do not sum to 100.**

**Solve missing number problems that sum to 100.**




62 + ☐ = 100

# Addition and Subtraction

## Year 3

## Columnar Addition (1)

**Vocabulary:**

Ones Tens Represents Compose Combine Total Dienes Plus + Minus - Equals = Is equal to = Addition Subtraction Equation Regroup Algorithm

**Addend (or augend) + Addend = Sum** **Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

***In column addition, we start at the right-hand side.***

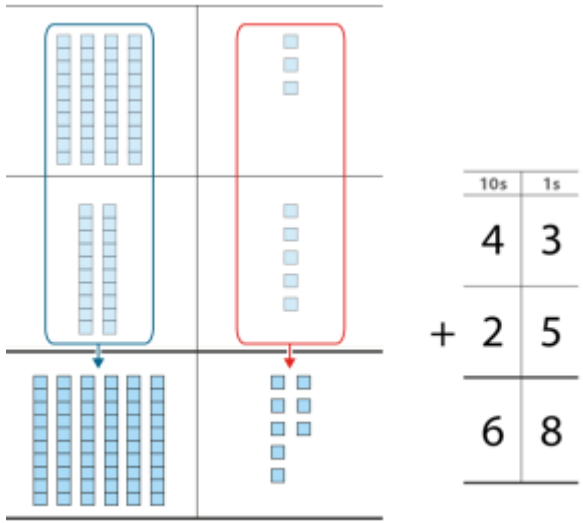
**Use Dienes to represent columnar addition *without regrouping* before moving to abstract algorithm.**

**We add the ones.**

**3 ones plus 5 ones is equal to 8 ones.**

**We add the tens.**

**4 tens plus 2 tens is equal to 6 tens.**



| | 10s | 1s |
|---|---|---|
| | 4 | 3 |
| + | 2 | 5 |
| | 6 | 8 |

Model moving all the pieces in a particular column down into the answer space to form the sum for that column.

Ensure children understand how the addends and sum are represented in column addition. Draw attention to the 'large equals symbol' that frames the sum.

Ensure that the manipulatives are used to highlight the structure, rather than to do the calculating; children should use known facts to find the sum of each column. They should not be counting the cubes to find the answer.

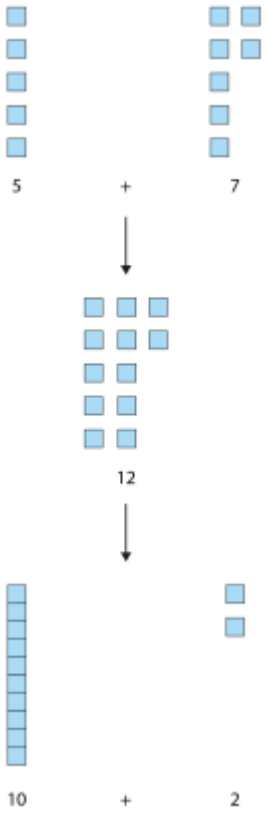
As children become familiar with how the algorithm works, remove the concrete apparatus.

Provide varied practice to include:

- + of three 2-digit numbers
- + of 3-digit numbers
- Cases where some of the digits are zero
- + of two number with different numbers of digits. Ensure the children can set these out correctly and align the digits.
- Calculations involving empty boxes in different positions

Ensure children can talk about what the digits represent within the algorithm.

In preparation for column addition with regrouping, give practice on regrouping teen numbers of ones into one ten and some ones.




Encourage children to describe the regrouping process in full using the language if unitizing:

*5 ones plus 7 ones is equal to 12 ones.*

*12 ones is equal to 1 ten and 2 ones.*

Also model the language of regroup:

*We can regroup 12 ones into 1 ten and 2 ones.*

# Addition and Subtraction

## Year 3

## Columnar Addition (2)

**Vocabulary:**

Ones Tens Represents Compose Combine Total Dienes Plus + Minus - Equals = Is equal to = Addition Subtraction Equation Regroup Algorithm Least/ most significant digit Align the digits

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

***In column addition, we start at the right-hand side.***

***If the column sum is ten or more, we must regroup.***

**Use Dienes to represent columnar addition *with regrouping* before moving to abstract algorithm.**

**5 ones plus 7 ones is equal to 12 ones. I can regroup 12 ones. 12 ones is equal to 1 ten and 2 ones.**

**2 tens plus 4 tens is equal to 6 tens. We also need to add 1 ten from the regrouping. There are 7 tens altogether.**

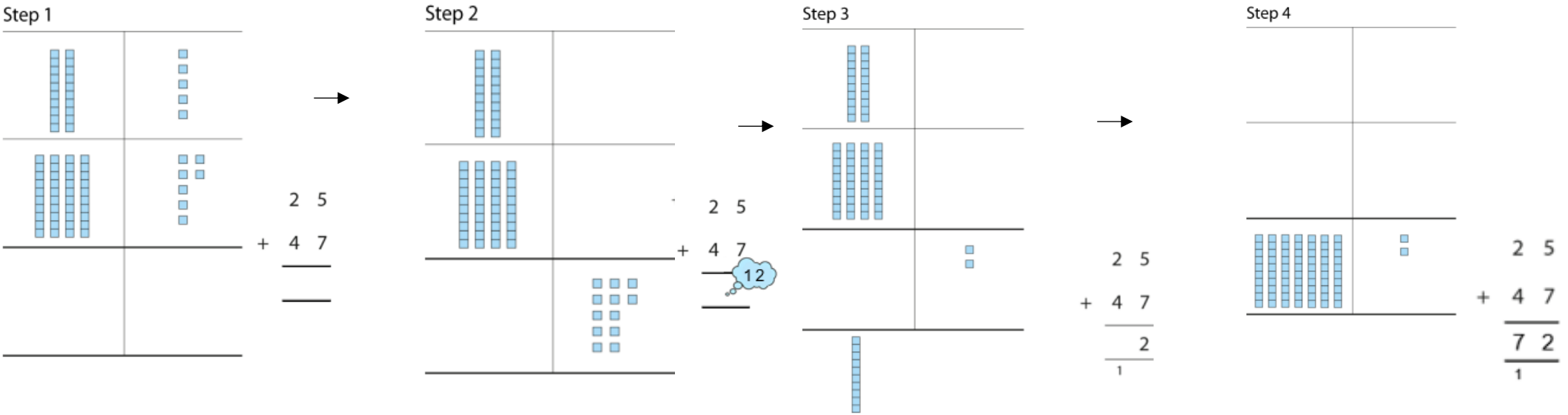
**If a column group is equal to 10 or more we must regroup. 10 ones is equal to 1 ten. 10 tens is equal to 1 hundred.**

When starting to regroup, start with a calculation where the ones digits sum to 10.

In Step 2, stress that we cannot record 12 in the ones column and reinforce the stem sentence: If the column sum is ten or more, we must regroup.

In Step 3, model how we record the regrouped digit underneath the answer space in the tens column, ready to add with the other tens.

Some children find it really helpful to cross out the regrouped digit as it's added to the other tens in the final step. It is good to model this.



# Addition and Subtraction

## Year 3

## Columnar Addition (3)

**Vocabulary:**
Ones Tens Represents Compose Combine Total Dienes Plus + Minus - Equals = Is equal to = Addition Subtraction Equation Regroup Algorithm Least/ most significant digit Align the digits

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

***In column addition, we start at the right hand side.***

***If the column sum is ten or more, we must regroup.***

Provide varied practice using the method above to add two 2-digit and 3-digit numbers where regrouping is needed in some or all columns. Also include:

- + of several addends which add to a number greater than 20 in column (eg 18 + 36 + 29) so children don't begin to believe that the regrouped digit is only ever 1.
- + of 2-digit number that sum to more than 100.
- Calculations involving empty boxes. Discuss: What could the missing number be? What can't it be?

It is essential that, once column methods are introduced, these do not become the default strategies and that children continue to engage their number sense and reasoning, making considered decisions about when mental methods are more appropriate.

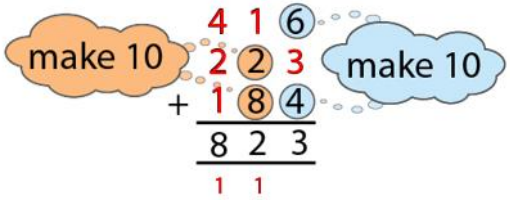
**Compare expressions which can be calculated using mental or written strategies.**

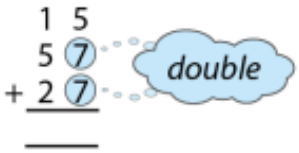
475 + 25 237 + 156

349 + 84 120 + 130

| Use column addition | Use mental strategies |
|---|---|
| | |

**Add 3 addends using columnar addition, using a make 10 strategy to support. Children should be able to choose the most efficient order to add digits within a column and use known facts, explaining their reasoning.**




**Use rules to check for errors quickly, justifying responses. Eg:**

$$\begin{array}{r} 650 \\ +\ 275 \\ \hline 935 \\ 1 \end{array} \times$$

*'The sum of two odd numbers is always an even number, so this can't be correct.'*

$$\begin{array}{r} 650 \\ +\ 275 \\ \hline 920 \\ 1 \end{array} \times$$

*'When zero is added to a number, the number remains the same, so this can't be correct.'*

$$\begin{array}{r} 935 \\ \hline 1 \end{array} \times$$

*'The sum of two odd numbers is always an even number, so this can't be correct.'*

$$\begin{array}{r} 920 \\ \hline 1 \end{array} \times$$

*'When zero is added to a number, the number remains the same, so this can't be correct.'*

Use other methods to encourage children to engage their number sense and to reason about the methods they are choosing/ using. Encourage estimation. See Year 3 Spine 1.20, p 18 & 19.

# Addition and Subtraction

## Year 3

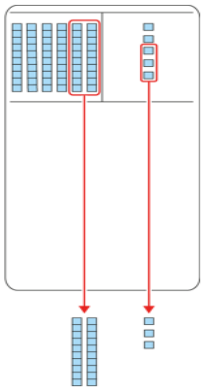
## Columnar Subtraction (1)

**Vocabulary:**

Ones Tens Represents Compose Combine Total Dienes Plus + Minus - Equals = Is equal to = Addition Subtraction Equation Expression Regroup Algorithm *Least/ most significant digit* *Align the digits*

**Addend + Addend = Sum** **Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***



| | 10s | 1s |
|---|---|---|
| | 6 | 5 |
| – | 2 | 3 |
| | 4 | 2 |

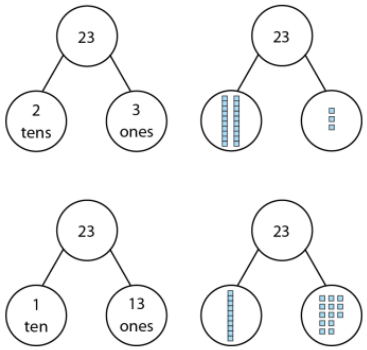
**Use Dienes to represent columnar subtraction *without regrouping* initially.**

**We subtract the ones. 5 ones minus 3 ones is equal to 2 ones.**

**We subtract the tens. 6 tens minus 2 tens is equal to 4 tens.**

Ensure children understand how the minuend, subtrahend and difference are represented in the algorithm. Draw attention to the 'large equals symbol' that frames the difference.

Introduce regrouping practically using Dienes so children become familiar with different representations of a number and are able to work flexibly in preparation for column subtraction.




23 is equal to 2 tens and 3 ones.

23 = 20 + 3 = 10 + 13

23 is also equal to 1 ten and 13 ones.

Move to introducing regrouping to solve a calculation when the ones digit in the minuend is smaller than the ones digit in the subtrahend. Eg 94 – 6

Solve first using Dienes and then record alongside each step.

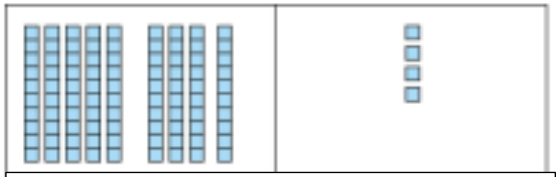
When recording the regrouping, show as here, crossing out the original number of tens and ones and recording the quantity of each after regrouping.

N.B. This is different to the NCETM, but has worked better for our children as it reinforces the new quantity of ones and reduces the chance of confusion with the small one written above the ones digit.

N.B. It should be stressed to the children that calculations like this should usually be done mentally. We are only doing this now as a step in learning the column method, which will be helpful for making tricky calculations with larger numbers.

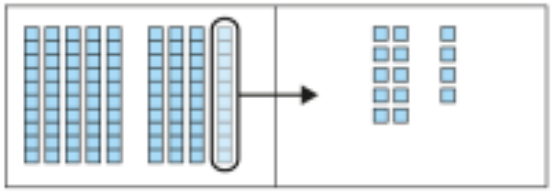
NB. We have decided to use 'regroup' rather than exchange, as used by the NCETM, as this links to +.

94 – 6



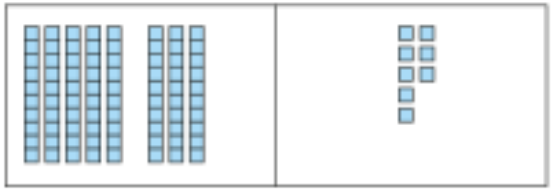
We cannot subtract 6 ones from 4 ones.

| | 10s | 1s |
|---|---|---|
| | 9 | 4 |
| – | | 6 |
| | | |



We regroup 9 tens and 4 ones into 8 tens and 24 ones.

| | 10s | 1s |
|---|---|---|
| | 9 (8) | 4 (14) |
| – | | 6 |
| | | |



14 ones subtract 6 ones is 8 ones.
8 tens subtract 0 tens is 8 tens.

| | 10s | 1s |
|---|---|---|
| | 9 (8) | 4 (14) |
| – | | 6 |
| | 8 | 8 |

# Addition and Subtraction

## Year 3

## Columnar Addition and Subtraction (2)

**Vocabulary:**

Ones Tens Represents Compose Combine Total Dienes Plus + Minus - Equals = Is equal to = Addition Subtraction Equation Expression Regroup Algorithm Least/ most significant digit Align the digits

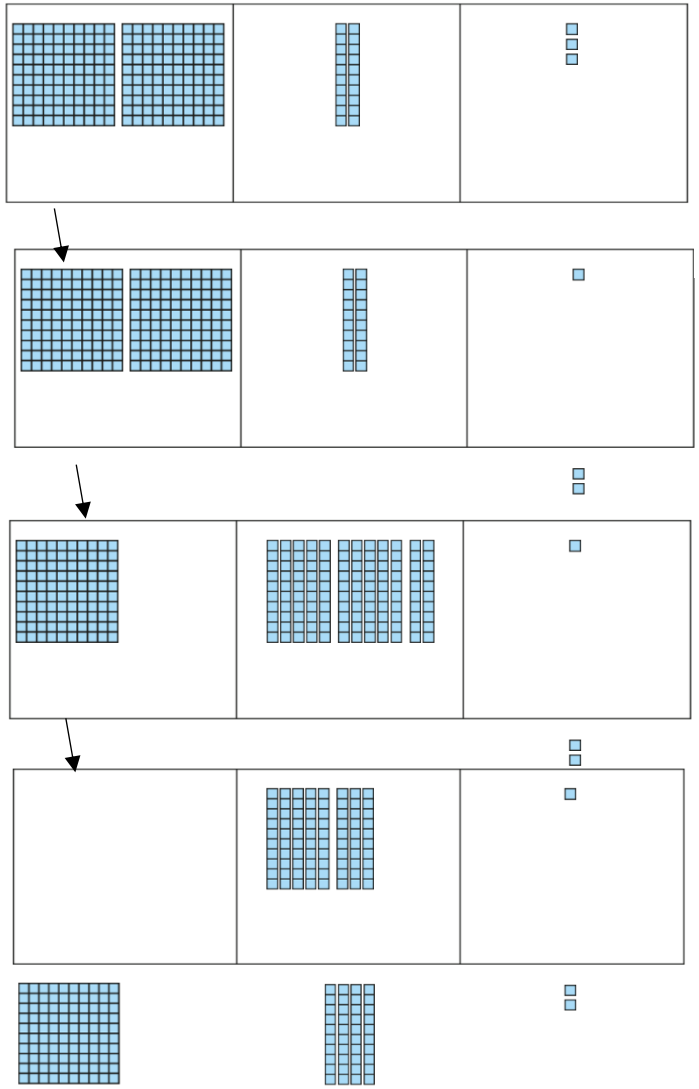
**Addend + Addend = Sum** **Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

With subtraction we only make the minuend with blocks and we subtract the subtrahend from this.

Move the subtracted blocks underneath as they are subtracted. They should remain visible so that we can make a clear link with the inverse operation to check the answer.

As with addition, include varied practice at all stages including examples where regrouping of the digits in different columns is needed and calculations with empty boxes.



The minuend has 2 hundreds, 2 tens and 3 ones.

| | | | |
|---|---|---|---|
| | 2 | 2 | 3 |
| − | 1 | 4 | 2 |
| | | | |

3 ones subtract 2 ones is equal to 1 one.

| | | | |
|---|---|---|---|
| | 2 | 2 | 3 |
| − | 1 | 4 | 2 |
| | | | 1 |

We cannot subtract 4 tens from 2 tens.

We must regroup.

We regroup 2 hundreds and 2 tens into 1 hundred and 12 tens.

| | | | |
|---|---|---|---|
| | 1 2 | 12 2 | 3 |
| − | 1 | 4 | 2 |
| | | | 1 |

12 tens subtract 4 tens is equal to 8 tens.

1 hundred subtract 1 hundred is zero hundreds.

223 subtract 142 is 81

| | | | |
|---|---|---|---|
| | 1 2 | 12 2 | 3 |
| | 1 | 4 | 2 |
| | 0 | 8 | 1 |

**Compare expressions which can be calculated using mental or written strategies.**

***The best mathematicians do the easiest maths!***

## Addition and Subtraction

## Year 3

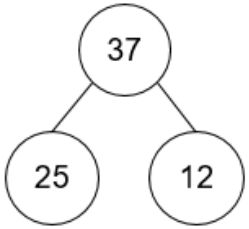
## Manipulate the Additive Relationship

**Vocabulary:**

Represents Compose Combine Total Dienes Plus + Minus -
Equals = Is equal to = Addition Subtraction Equation Expression Bar Model
Part-Part-Whole Model Whole Part

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**
***We subtract the subtrahend.***




| 37 | |
|---|---|
| 25 | 12 |

25 + 12 = 37
12 + 25 = 37

37 = 25 + 12
37 = 12 + 25

37 – 12 = 25
37 – 25 = 12

25 = 37 – 12
12 = 37 – 25

**Recognise the different equations that can be recorded based on the part-whole structure.**

**Addend + addend = sum**

**Minuend – subtrahend = difference**

Part Part Whole

329 + 414 = 743

| 743 | |
|---|---|
| 329 | 414 |

743 – 329 = 414

Whole Part Part

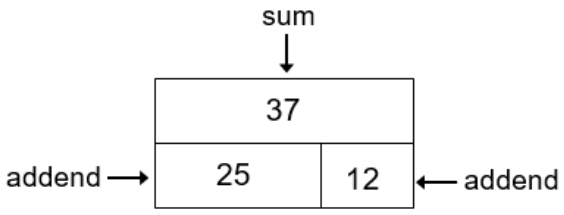
447 – 162 = 285

| 447 | |
|---|---|
| 285 | 162 |

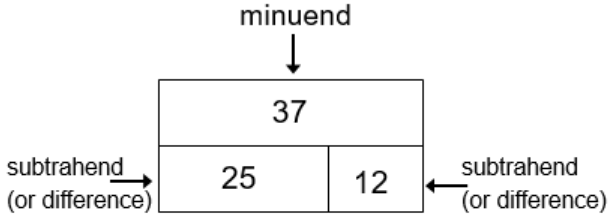
447 – 285 = 162

**Use the part-whole structure to support finding a missing part.**

**There is a missing part. To find the missing part, we subtract the other part from the whole.**




25 + 12 = 37
12 + 25 = 37




37 – 25 = 12
37 – 12 = 25

Whole Part Part

614 – 527 = 87

| 614 | |
|---|---|
| 527 | 87 |

527 + 87 = 614

**Use the part-whole structure to support finding a missing whole.**

**There is a missing whole. To find the missing whole, we add the two parts.**

# Addition and Subtraction

## Year 4 and 5

## Columnar Addition and Subtraction

**Vocabulary:**

Ones Tens Represents Compose Combine Total Dienes Plus + Minus - Equals = Is equal to = Addition Subtraction Equation Expression Regroup Algorithm Least/ most significant digit Align the digits

**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

In Years 4 and 5, children build on their use of columnar methods to add and subtract a wider range of numbers. Refer to the steps, vocabulary and stem sentences detailed for Year 3. Ensure work continues to be done to reinforce mental strategies and promote number sense.

Ensure that when a new range of numbers is introduced, manipulatives (Dienes/ Place Value counters) are used to support children's understanding of structure and remove when ready. Children should not be using manipulatives to do the calculation.

Year 4:

- Composition of 1000
- Addition and subtraction of 4-digit numbers
- Addition and subtraction of numbers involving tenths, hundredths and thousandths.
- Addition and of money.

Year 5:

- Use columnar and mental methods to:
  - Add and subtract 5 and 6-digit numbers
  - Continue to add and subtract numbers involving tenths, hundredths and thousandths, including money and measures.
- Count, compare and calculate with negative numbers
- Use equivalence and the compensation properties to calculate
  1. If one addend is increased and the other is decreased by the same amount, the sum stays the same
  2. If one addend is increased (or decreased) and the other stays the same, the sum increases (or decreases) by the same amount.
  3. If the minuend and subtrahend are changed by the same amount, the difference stays the same (same difference).
  4. If the minuend is increased (or decreased) and the subtrahend is kept the same, the difference increases (or decreases) by the same amount.
  5. If the minuend is kept the same and the subtrahend is increased (or decreased) the difference decreases (or increases) by the same amount.
  6. The value of the expressions on each side of an equals symbol must be the same; addition and subtraction are inverse operations, we can use this knowledge to balance equations and solve problems.

## Addition and Subtraction

## Year 6

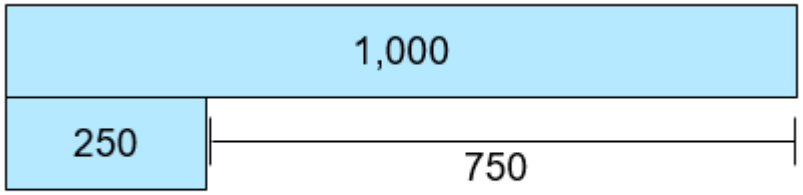
## Quantify additive and multiplicative relationships

**Vocabulary:**
Additive Multiplicative Relationship Represents Compose Combine Total More than Less than Plus + Minus - Equals = Is equal to = Addition Subtraction Divide ÷ Multiply x ___ groups of ___ Equation Expression Bar Model Whole Part Difference Multiplier Unknown Sequence

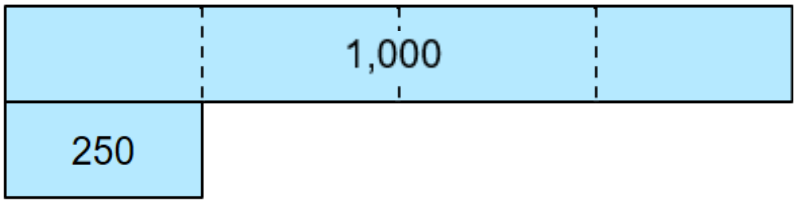
**Addend + Addend = Sum**

**Minuend – Subtrahend = Difference**

***We subtract the subtrahend.***

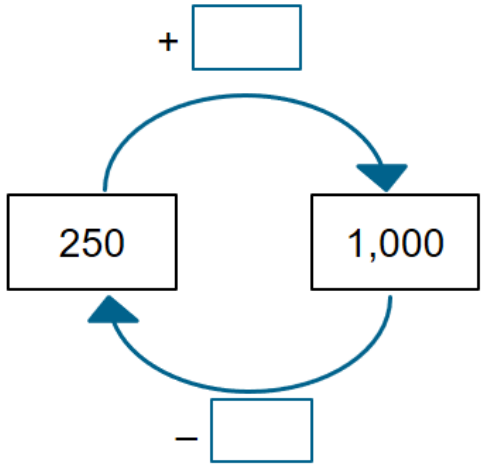



250 + 750 = 1,000 1,000 – 750 = 250



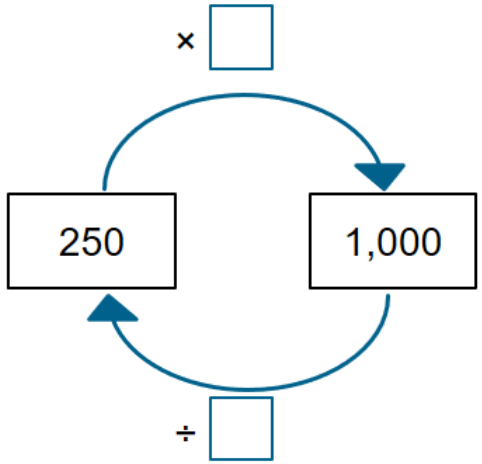

250 x 4 = 1,000 1000 ÷ 4 = 250

**The relationship between two numbers can be expressed**




**1000 is ___ more than 250.**

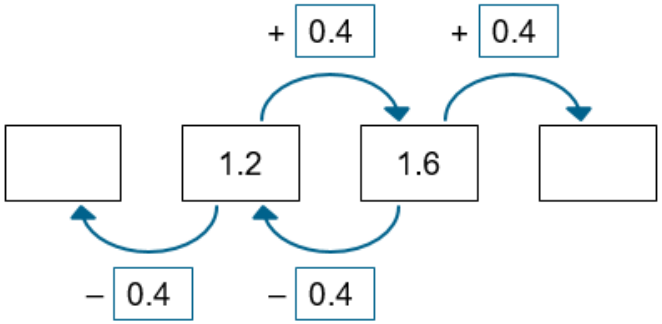
**250 is ____ less than 1000.**




**1000 is ___ times the size of 250.**

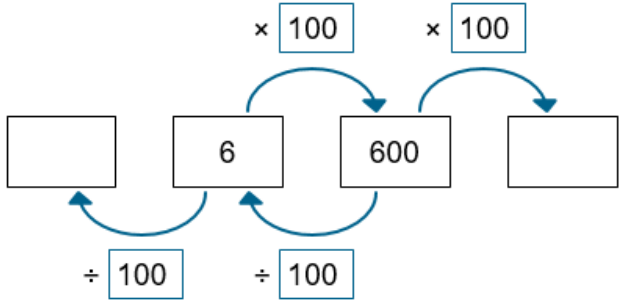
**250 is one-_____ of 1000.**

**To find one-quarter of a number, we divide by 4.**




**Finding the difference can help calculate the unknown terms in a sequence.**

**Finding the known multiplier can help calculate the unknown terms in a sequence.**



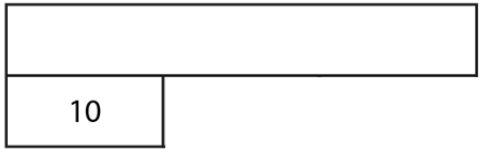
## Addition and Subtraction

## Year 6

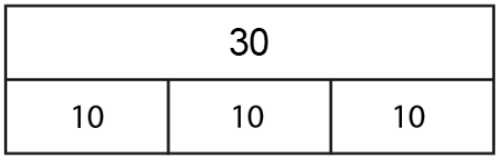
## Quantify additive and multiplicative relationships

**Vocabulary:**
Additive Multiplicative Relationship Represents Compose Combine Total
More than Less than Plus + Minus - Equals = Is equal to =
Addition Subtraction Divide ÷ Multiply x One-_____ of Equation Expression
Bar Model Whole Part Difference Multiplier Unknown Sequence

$\frac{1}{3}$ of ?= 10




$\frac{1}{3}$ of ?= 10




$\frac{1}{3}$ of 30 = 10

**Calculate the unknown whole by recognising how many parts the whole has been divided into.**

## Addition and Subtraction

## Year 6

## Derive Related Calculations

**Vocabulary:**

Additive Multiplicative Relationship Represents Equation Unknown Re-arrange Inverse Place Value Properties Commutative Associative Distributive Compensation

**Addend + Addend = Sum Factor x Factor = Product (Multiplicand x Multiplier = Product)**
**Minuend – Subtrahend = Difference Dividend ÷ Divisor = Quotient**

| | | |
|---|---|---|
| $252 = 3 \times 84$ | $252 = 3 \times 84$ | $252 = 3 \times 84$ |
| $2{,}520 = 30 \times \square$ | $\square = 3 \times 85$ | $252 = 3 \times 60 + 3 \times \square$ |

| | | |
|---|---|---|
| $625 - 148 = 477$ | $625 - 148 = 477$ | $625 - 148 = 477$ |
| $6{,}250 - 1{,}480 = \square$ | $625 - 70 - \square = 477$ | $625 - 248 = \square$ |

| | | |
|---|---|---|
| $14.8 + 7.6 = 22.4$ | $14.8 + 7.6 = 22.4$ | $14.8 + 7.6 = 22.4$ |
| $1{,}480 + \square = 2{,}240$ | $\square - 7.6 = 14.8$ | $12.8 + \square = 22.4$ |

| | | |
|---|---|---|
| $4{,}800 \div 25 = 192$ | $4{,}800 \div 25 = 192$ | $4{,}800 \div 25 = 192$ |
| $25 \times 192 = \square$ | $4{,}800 \div 250 = \square$ | $4{,}800 \div 5 \div 5 = \square$ |

**Manipulate an equation to solve another. Pupils could:**

- **rearrange the terms;**
- **rewrite using inverse operations;**
- **apply place value;**
- **use the properties of division that correspond to the commutative, associative or distributive property of multiplication;**
- **use the compensation property.**

**Additive examples**

**Multiplicative examples**

## Addition and Subtraction

## Year 6

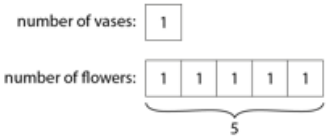
## Solve Problems involving Ratio Relationship

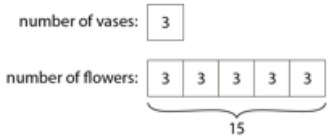
**Vocabulary:**

Additive Multiplicative Relationship Represents Equation Unknown Scale-factor Ratio Ratio Table ___ times the size one-___ the size of Vertical Horizontal

**Factor x Factor = Product (Multiplicand x Multiplier = Product)**

**Dividend ÷ Divisor = Quotient**




$1 \times \mathbf{5} = 5$

$5 \div \mathbf{5} = 1$

$5 \times \frac{1}{5} = 1$

$3 \times \mathbf{5} = 15$

$15 \div \mathbf{5} = 3$

$15 \times \frac{1}{5} = 3$

**Ratio table to compare sets of information.**

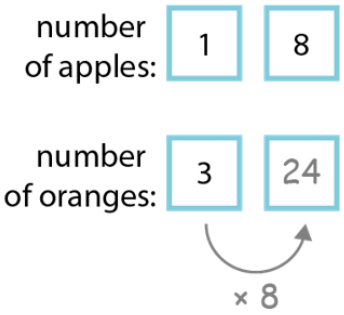
**For every ___, there are ___.**

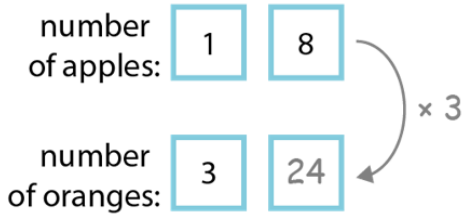
**For every 1 litre of petrol, you can drive 7 miles.**

**For every 7 miles you will drive, you need 1 litre of petrol.**

**Extend sequences using knowledge of patterns based on ratio table.**

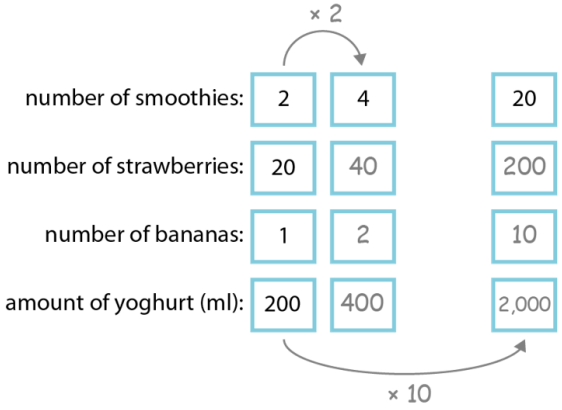
| Litres of petrol | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Miles driven | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |




**Explore vertical and horizontal relationship between numbers.**

**For every ___, there are ___.**

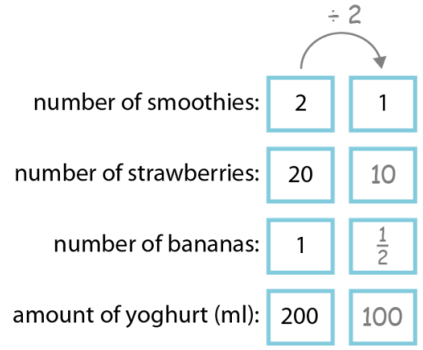



**Identify the scale-factor in order to find unknown values.**

**___ is ___ times the size of ___.**

**Therefore I must multiply/divide by ___.**

**___ is one-__ the size of ___.**



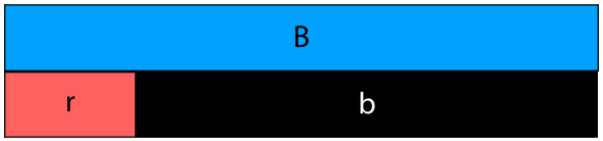
## Addition and Subtraction

## Year 6

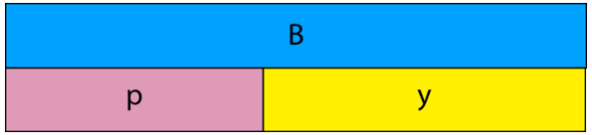
## Solve Problems with Two Unknowns

**Vocabulary:**

Additive Multiplicative Relationship Represents Equation Two Unknowns Scale-factor Ratio ___ times the size one-___ the size of Total Bar Model Structure




$B = r + b$




$B = p + y$

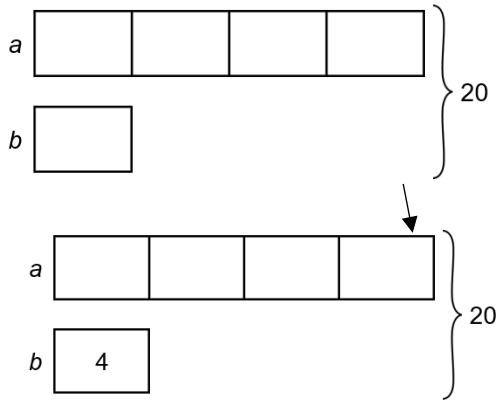
**Use Cuisenaire to find 2 bars of total length that are equal to another.**

**There is more than one solution to the problem.**

**There can be infinite solutions to a problem.**

$5 \times \square = 10 \times \square$

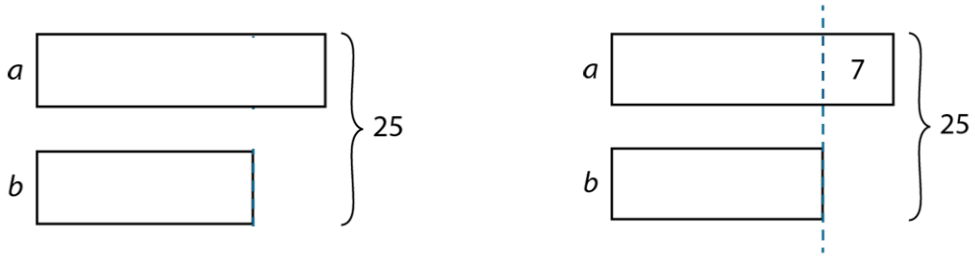
**Solve multiplicative problems with two unknowns when the total is known.**

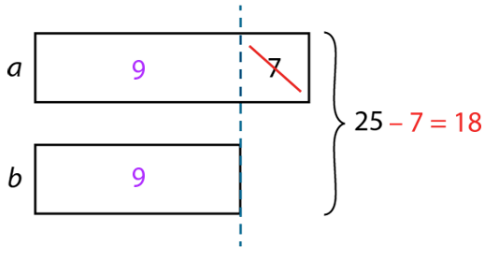



one part = 20 ÷ 5 = 4

*b* = 4

**Solve additive problems with two unknowns when the total is known.**

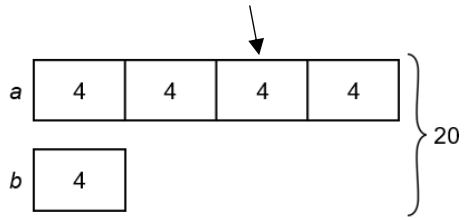



*b* = 18 ÷ 2 = 9

*a* = 9 + 7 = 16

*The two numbers are 9 and 16.*




one part = 20 ÷ 5 = 4

*b* = 4

*a* = 4 × 4 = 16

*The two numbers are 16 and 4.*

# Multiplication and Division

# Multiplication and Division

## Year 2

## Multiplication as Repeated Addition

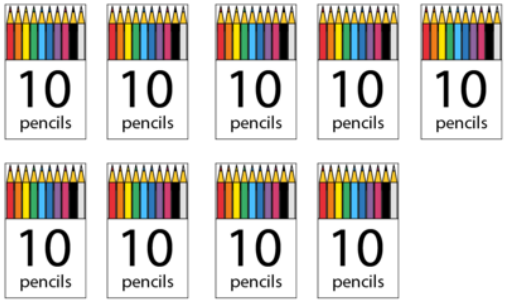
**Vocabulary:**

Group Equal Unequal Repeated Addition Multiplication Expression Equation Part Altogether Represents Amount Size

Factor Product

**Understand the difference between equal and unequal groups.**

**The ___ have been grouped into equal/ unequal groups.**




**Notice how the representations allow the children to see each of the numbers/ quantities (i.e. 10 pencils and 9 packets).**

**9 × 10**

**We can represent equal groups as repeated addition.**

**There are 3 groups of 5.**

**5 + 5 + 5**
**3 × 5**

**5 + 5 + 5 = 3 × 5**

**We can skip count in multiples of __ to work out the total amount.**

**10, 20, 30, 40 … there are 90 pencils altogether.**

**We can represent repeated addition using a multiplication expression.**

**The 3 represents the number of groups.**

**The 5 represents the number of eggs in each group.**

**15 represents the total number of eggs.**

**Ensure children understand what the numbers represent in expressions and equations and can relate these to real contexts.**

**The ___ represents the number of groups.**

**The __ represents the number of ___ in each group.**

**___ represents the total number of ___.**

**7 × 2**

# Multiplication and Division

## Year 2

## Grouping problems: missing factors and division

**Vocabulary:**

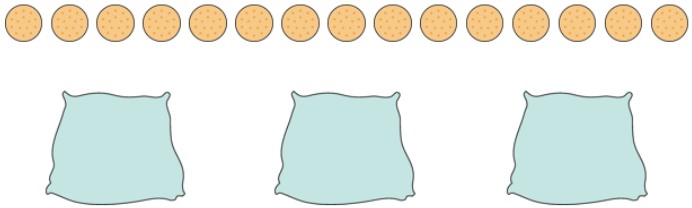
Multiplication  Division  Factor  Product  Represents  Skip Counting

Groups  Amount  Size  'divided by'

Explore putting quantities of objects into equal groups as a lead in to division.

Discuss different ways of grouping, eg 15 is equal to 3 groups of 5.

Express as an equation:  15 = 3 x 5



$\boxed{3} \times 5 = 15$

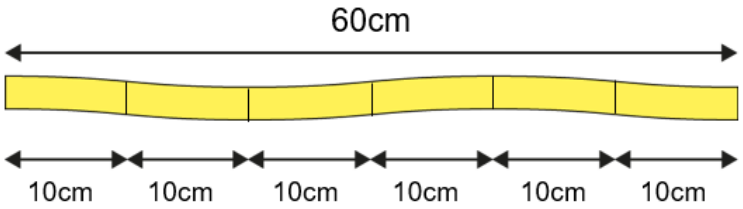
$15 \div 5 = \boxed{3}$

**We can solve division problems by finding missing factors.**

**The 15 represents the number of biscuits.**

**The 5 represents the number of biscuits in each bag (group).**

**The 3 represents the number of bags (groups).**




$\boxed{6} \times 10 = 60$

$60 \div 10 = \boxed{6}$

**The 60cm represents the length of the ribbon.**

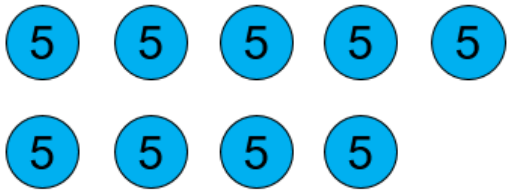
**The 10 represents the size of each piece.**

**The 6 represents the number of pieces we can make.**

**We can use ÷ to mean 'divided by'**

**We can use our knowledge of times tables to help solve division problems.**

$45 \div 5 = \boxed{9}$



# Multiplication and Division

## Year 3

## Multiplication and Division Structures (1)

## Manipulating the Multiplicative Relationship

**Vocabulary:**

Multiplication Division Commutative Grouping (Quotitive) Sharing (Partitive) 'Divided into' 'Divided between' 'Divided by' Equation Expression Factor Product Ratio table

The multiplicand is the size of each group. The multiplier is the number of groups.

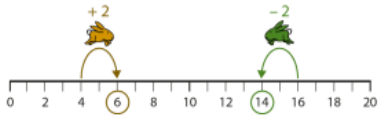
**Introduction to the distributive property of multiplication: adjacent multiples of 2 have a difference of 2. This applies to all multiples patterns.**

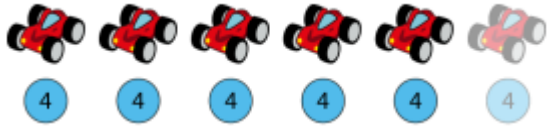
Adjacent multiples of two have a difference of two:

| | ×2 |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | |
| 4 | 8 |
| 5 | 10 |
| 6 | 12 |
| 7 | |
| 8 | 16 |
| 9 | 18 |
| 10 | 20 |
| 11 | 22 |
| 12 | 24 |

↓+2 $3\times2=2\times2+2$

↑−2 $7\times2=8\times2-2$




| | |
|---|---|
| $0\times4=0$ | $4\times0=0$ |
| $1\times4=4$ | $4\times1=4$ |
| $2\times4=8$ | $4\times2=8$ |
| $3\times4=12$ | $4\times3=12$ |
| $4\times4=16$ | $4\times4=16$ |
| $5\times4=20$ | $4\times5=20$ |
| **$6\times4=24$** | **$4\times6=24$** |

**Ratio Table**

| Number of cars | Total number of wheels |
|---|---|
| 0 | 0 |
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 4 | 16 |
| 5 | 20 |
| **6** | **24** |

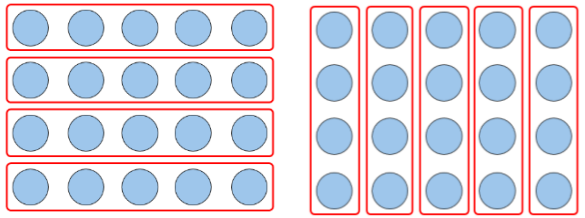
**Introduction to ratio tables**

Pose questions such as:

*How many wheels do 4 cars have?*

*How many cars are there if there are 24 wheels?*

*What are the factors of 8?*



**Reinforce that multiplication is commutative.**

**4 x 5 = 5 x 4**

**Factor times factor is equal to product.**

**The order of the factors does not affect the product.**

**Explore the relationship between multiplication tables,** eg

The products in the 4 times table are double the products in the 2 times table.

Represent using a range of models.

Explore the inverse: Products in the 2 times table are half of those in the 4 times table.

This extends to the 8x table and links between other times tables are made later using the same ideas.

**Multiplicand & Multiplier**

If there is a context for the multiplication, we can use these terms to identify the role of each number.

The multiplicand is the size of each group. The multiplier is the number of groups.

These words, although not featured in the NCETM materials, can help us to explain and explore multiplication. For example, exploring the effect on the product of increasing the multiplicand by one and how this is different to increasing the multiplier by one, will deepen children's understanding of multiplication and can support with later learning.

**Deepen understanding of distributive law and multiplicative structure using empty boxes**

$3\times4=2\times4+\square$

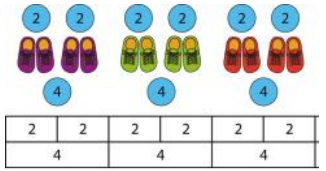
$3\times4-\square=2\times4$

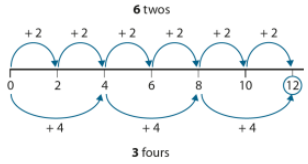
*'Fill in the missing symbols (<, > or =).'*

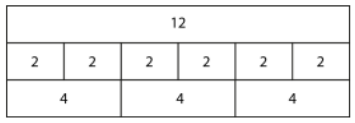
$9\times4 \bigcirc 8\times4$

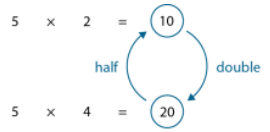
$9\times4 \bigcirc 8\times4+4$

**The Numberlink Board is a great tool to support the learning of multiplication facts, explore connections between times tables and the distributive law.**



# Multiplication and Division

## Year 3

## Multiplication and Division Structures (2)

**Vocabulary:**

Multiplication Division Commutative Grouping (Quotitive) Sharing (Partitive)
'Divided into' 'Divided between' 'Divided by' Equation Expression
Factor Product
Dividend Divisor Quotient

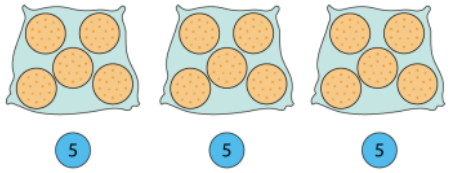
| | | | | |
|---|---|---|---|---|
| 30 | ÷ | 5 | = | 6 |
| dividend | ÷ | divisor | = | quotient |

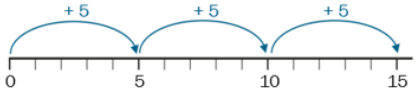
Explore arranging quantities of objects into equal groups as a lead in to division.

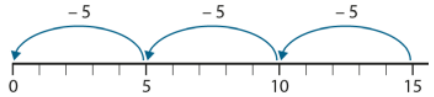
Do all numbers make equal groups?

Discuss different ways of grouping, eg 15 is equal to 3 groups of 5.
Express as an equation: 15 = 3 x 5. What does each number represent?

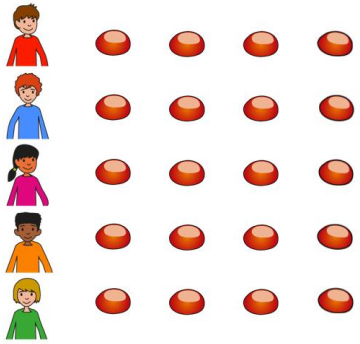
NB. This is not covered in NCETM materials, but is helpful to deepen children's understanding and make connections between x and ÷ and supports learning in Y4 re. remainders, bridging the work done in Y2.
Explore empty boxes, eg 15 = ☐ x 5
"15 is equal to * groups of 5."




**Making groups of**



Focus on repeated addition when dividing. This will ensure that children use their knowledge of skip counting in multiple groups.

Counting back becomes problematic when a remainder is involved. (Y4)

**Division equations can be used to represent 'grouping' problems.**

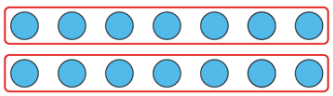
**We can use multiplication facts to find the number of groups.**

**(Quotitive division)**

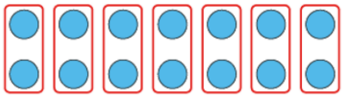
**15 divided into groups of 5 is equal to 3 in each group.**
**5 + 5 + 5 = 15**
**15 – 5 - 5 – 5 = 0**
**15 ÷ 5 = 3**

**The same equation can be represented in both grouping and sharing contexts.**

**Explore how the bar model looks different.**



14 ÷ 2 = 7



14 ÷ 2 = 7

| 14 | |
|---|---|
| 7 | 7 |

| 14 | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |

**Division equations can be used to represent 'sharing' problems.**

**We can use multiplication facts to find the size of groups.**

**(Partitive division)**

**Four fives are four each.**
**20 divided between 5 is equal to 4 each.**
**20 ÷ 5 = 4**

**The Numberlink Board is a great tool to support the learning of multiplication and division facts, explore the distributive law and make connections.**

# Multiplication and Division

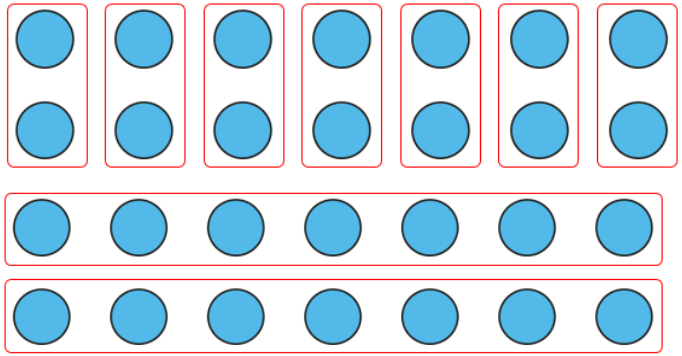
## Year 4

## Manipulating the Multiplicative Relationship

**Vocabulary:**

Multiply Divide Commutative Groups of Times Equal to Factors
Product Quotient Dividend Divisor Represents Array Ratio table

The multiplicand is the size of each group. The multiplier is the number of groups.



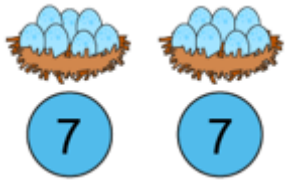
$2 \times 7 = 7 \times 2$

**Understand that multiplication is commutative and the factors can be**

***2 groups of 7 is equal to 14.***

***2, 7 times is equal to 14.***

***2 groups of 7 is equal to 7, two times.***




$2 \times 7 = 14$

$7 \times 2 = 14$

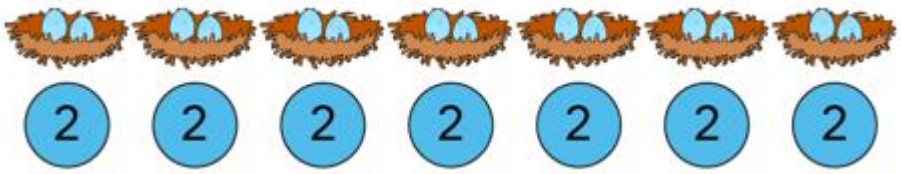
***The 2 represents ___.***

***The 7 represents ___.***

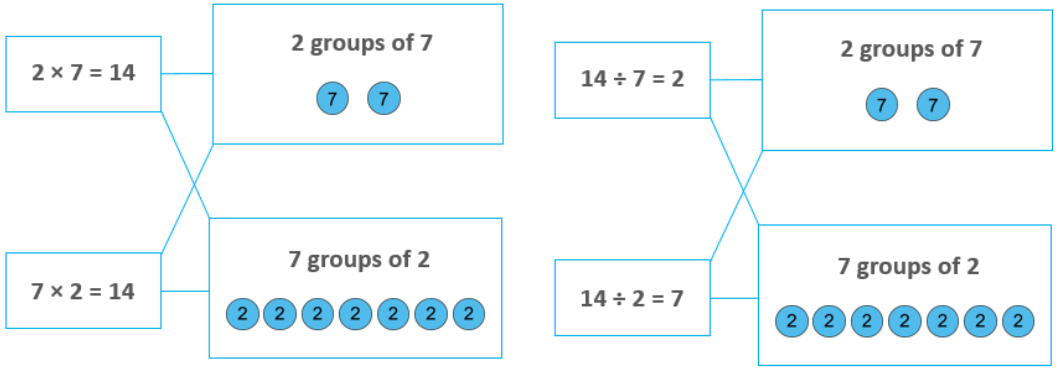
***The 14 represents ___.***

**Multiplicand & Multiplier**

If there is a context for the multiplication, we can use these terms to identify the role of each number.
The multiplicand is the size of each group. The multiplier is the number of groups.
These words, although not featured in the NCETM materials, can help us to explain and explore multiplication. For example, exploring the effect on the product of increasing the multiplicand by one and how this is different to increasing the multiplier by one, will deepen children's understanding of multiplication and can support with later learning.




**Match equations to representations and contexts.**



# Multiplication and Division

## Year 4

## The Distributive Property of Multiplication

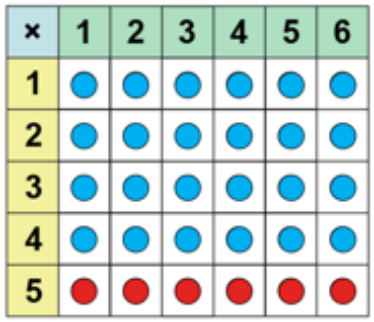
**Vocabulary:**

Multiplication Distributive Law Adjacent Multiples Factors Partitioning Equations Expressions Arrays Part-whole model Difference

The multiplicand is the size of each group. The multiplier is the number of groups.

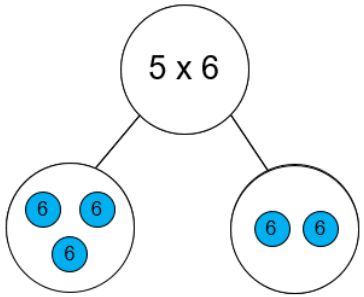
| | |
|---|---|
| 0 × 6 = 0 | 6 × 0 = 0 |
| 1 × 6 = 6 (+ 6) | 6 × 1 = 6 (+ 6) |
| 2 × 6 = 12 (+ 6) | 6 × 2 = 12 (+ 6) |
| 3 × 6 = 18 (+ 6) | 6 × 3 = 18 (+ 6) |
| 4 × 6 = 24 (+ 6) | 6 × 4 = 24 (+ 6) |
| 5 × 6 = 30 (+ 6) | 6 × 5 = 30 (+ 6) |
| 6 × 6 = 36 | 6 × 6 = 36 |
| 7 × 6 = 42 | 6 × 7 = 42 |
| 8 × 6 = 48 | 6 × 8 = 48 |
| 9 × 6 = 54 | 6 × 9 = 54 |
| 10 × 6 = 60 | 6 × 10 = 60 |
| 11 × 6 = 66 | 6 × 11 = 66 |
| 12 × 6 = 72 | 6 × 12 = 72 |

**Adjacent multiples of ___ have a difference of ___.**



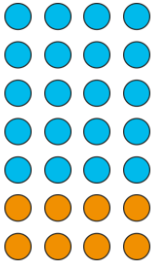

**4 × 6 + 6**

**Five sixes is one more six than four sixes.**




3 x 6 + 2 x 6 = 5 x 6

**5 is equal to 3 plus 2, so 5 sixes is equal to 3 sixes plus 2 sixes.**



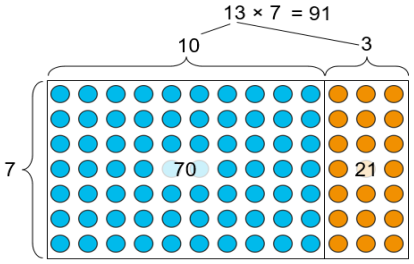
**7 = 5 + 2**

7 × **4** = 5 × **4** + 2 × **4**

= 20 + 8

= 28

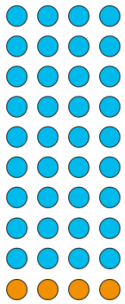
**We can partition one of the factors to make calculations easier.**




13 × 7 = 10 × 7 + 3 × 7

= 70 + 21

= 91



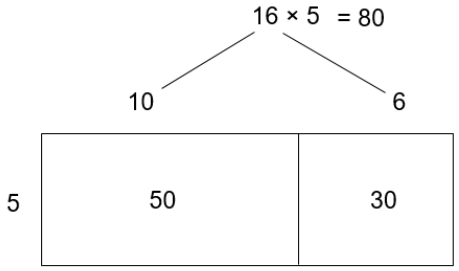
**9 = 10 − 1**

9 × 4 = **10** × 4 − **1** × 4

= 40 − 4

= 36

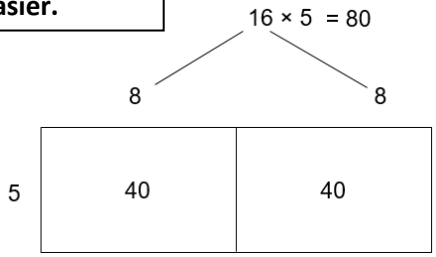
**We can partition the factors in different ways to make calculations easier.**




16 × 5 = 10 × 5 + 6 × 5

= 50 + 30

= 80




16 × 5 = 8 × 5 + 8 × 5

= 40 + 40

= 80

**The Numberlink Board is a great tool to explore the distributive law.**

# Multiplication and Division

## Year 4

## Division and remainders

**Vocabulary:**

Multiplication Division Commutative Grouping (Quotitive) Sharing (Partitive)
'Divided into' 'Divided between' 'Divided by' Equation Expression
Factor Product
Dividend Divisor Quotient
Remaining Remainder

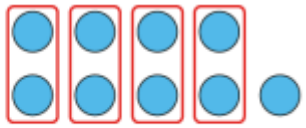
| | | | | | | |
|---|---|---|---|---|---|---|
| 14 | ÷ | 4 | = | 3 | r | 2 |
| dividend | ÷ | divisor | = | quotient | r | remainder |

Explore arranging quantities into equal groups and express using a multiplication equation, eg 8 = 4 x 2 *('8 is equal to 4 groups of 2.')*

Explore what the numbers represent.
**The 8 represents the total number of counters.**
**The 4 represents the 4 groups. The 2 represents the number of counters in each group,**

Explore a quantity that cannot be partitioned into equal groups, eg 9. Express as an equation:

.



$9 = 4 \times 2 + 1$

***Nine is divided into groups of 2. There are four groups of 2 and a remainder of 1.***

Explore what the numbers represent.
**The 9 represents the total number of counters.**
**The 4 represents the 4 groups. The 2 represents the number of counters in each group,**
**The 1 represents the remaining one counter.**

**Provide lots of practice of grouping counters and expressing in this way.**

Build on from Y3 work on quotitive and partitive division – real contexts, with and without remainders.
(See p35 above.) NB: The NCETM models division by adding and subtracting groups on a number line. When there is a remainder, subtracting can becomes error prone as it does not utilise children's knowledge of multiple patterns. It is helpful to show this strategy to explore how the remainder is represented, but children should not spend time practising this.

Through intelligent practice, children will explore when division will result in a remainder and when it won't, and how the divisor will affect the size of the remainder.

**Stem sentences:**

*'__ is a multiple of __ , so when it is divided into equal groups of __ there are none left over; there is no remainder.'*

*'__ is not a multiple of __, so when it is divided into equal groups of __ there are some left over; there is a remainder.'*

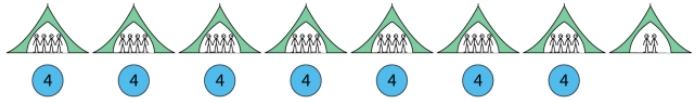
Introduce children to a variety of contexts where they need to make sense of the remainder to find the solution to a problem, either by 'rounding' the quotient up or down.

*'Four scouts can fit in each tent. How many tents will be needed for thirty scouts?'*


$30 \div 4 = 7$ r 2

- *'The "30" represents the total number of scouts.'*
- *'The "4" represents the number of scouts in each tent.'*
- *'The "7" represents the number of full tents.'*
- *'The "2" represents the number of scouts left over.'*

- *'We need another tent for the two left-over scouts. Eight tents are needed.'*

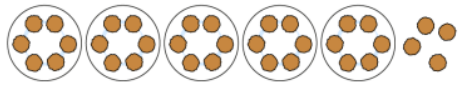



Generalisations

**If the dividend is a multiple of the divisor, there is no remainder.**
**If the dividend is not a multiple of the divisor, there is a remainder.**
**The remainder is always less than the divisor.**

**The Numberlink Board is a great tool to support the learning of division facts and explore remainders.**

*'Stephanie is having a party. She has thirty-four biscuits and wants to put them onto plates of six. How many full plates of six can she make?'*



$34 \div 6 = 5$ r 4

- *'The "34" represents the total number of biscuits.'*
- *'The "6" represents the number of biscuits on each plate.'*
- *'The "5" represents the number of plates of biscuits.'*
- *'The "4" represents the number of biscuits left over.'*

- *'So, five full plates of biscuits can be made.'*

# Multiplication and Division

## Year 4

## Multiplying and Dividing by 10 and 100

**Vocabulary:**

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1,000 | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 | 9,000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1,000 | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 | 9,000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**8 made ___ times the size is ___.**

**Develop language in order to multiply and divide by 10 or 100.**

**80 is ten times bigger than 8.**
**8 is ten times smaller than 80.**
**80 is ten times the size of 8**
**8 is one-tenth the size of 80.**

**800 is one hundred times bigger than 8.**
**8 is one hundred times smaller than 800.**
**800 is on hundred times the size of 8**
**8 is one-hundredth the size of 80.**

**8 x 1 = 8**
**8 x 1 ten – 8 tens**
**8 x 1 hundred = 8 hundreds**

**Generalisations**

**All multiples of 10 have a ones digit of zero.**

**All multiples of 100 have both a tens and ones digits of zero.**

**To find the inverse of ___times as many, you divide by ____.**

**If one factor if made ___ times bigger/smaller then the product will be ten times bigger/smaller**

ten times the size
x10

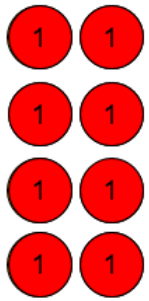
| 1,000s | 100s | 10s | 1s |
|---|---|---|---|
| | | | 8 |
| | | 8 | 0 |

÷10
one-tenth of the size

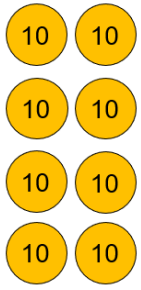
one hundred times the size
x100

| 1,000s | 100s | 10s | 1s |
|---|---|---|---|
| | | | 8 |
| | 8 | 0 | 0 |

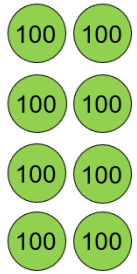
÷100
one-hundredth of the size




**8 × 1 = 8**




**8 × 10 = 80**




**8 × 100 = 800**

**8 groups of ___ is ___.**

# Multiplication and Division

## Year 5

## Multiplying and Dividing by 10 and 100 (1)

**Vocabulary:**

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse Ones Tens Hundreds Tenths Hundredths

**8 ÷ 10 =**

**0.8 ÷ 10 =**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1,000 | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 | 9,000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |

÷ 10
÷ 10
one-tenth the size

**0.08 x 10 =**

**0.8 x 10 =**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1,000 | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 | 9,000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |

× 10
× 10
ten times the size

**We can multiply and divide a number by 10.**

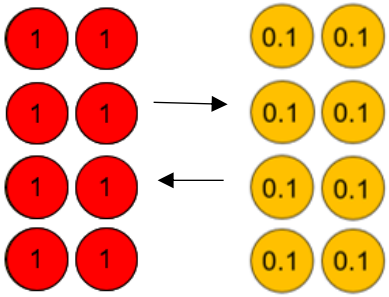
***8, made one-tenth the size is 0.8.***

***8 divided by 10 is 0.8.***

***First we had 8 ones, now we have 8 tenths.***

**8 ÷ 10 = 0.8**

one-tenth of the size




**0.8 x 10 = 8**

ten times the size

**We can multiply and divide a number by 100. Multiplying by 100 is the same as multiplying/dividing by 10 twice.**

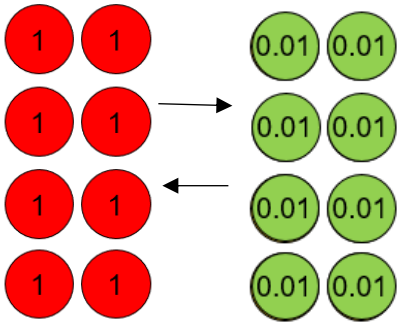
***8, made 100 times smaller is 0.08.***

***8 divided by 100 is 0.08.***

***First we had 8 ones, now we have 8 hundredths***

**8 ÷ 100 = 0.08**

one-hundredth of the size




**0.08 x 100 = 8**

one hundred times the size

## Multiplication and Division

## Year 5

## Multiplying and Dividing by 10 and 100 (2)

**Vocabulary:**

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse Ones Tens Hundreds Tenths Hundredths

**3.6 × 10 = 36**

**36 ÷ 10 = 3.6**

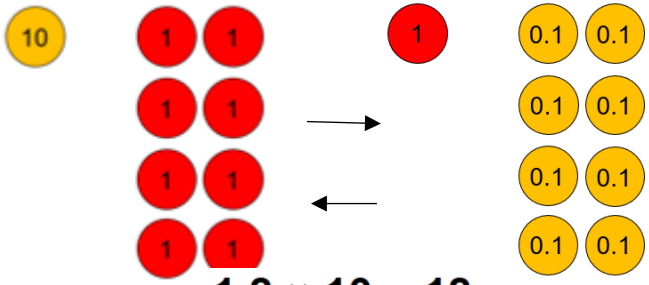
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1,000 | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 | 9,000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |

**18 ÷ 10 = 1.8**

one-tenth of the size

*1.8 is one-tenth the size of 18*

*18 divided by 10 is 1.8.*




**1.8 × 10 = 18**

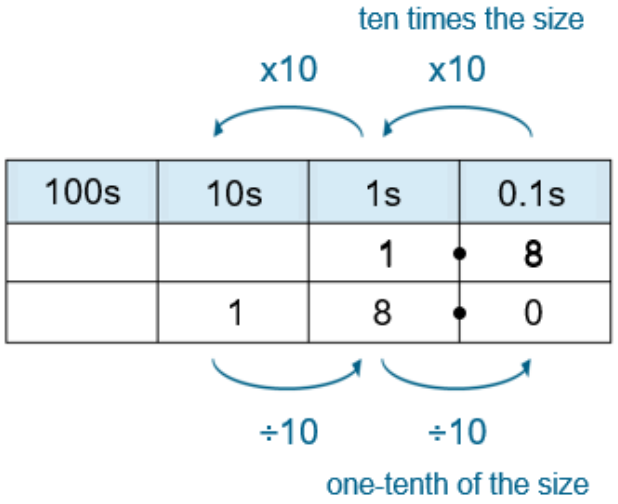
ten times the size

*__ divided by 10/100 is equal to__.*

*__ is one-tenth/hundredth the size of __.*

*__ multiplied by 10/100 is equal to__.*

*__ is 10/100 times the size of __.*




**Generalisation**

*To multiply by 10, move each digit one place to the left.*

*To multiply by 100, move each digit two places to the left.*

*To divide by 10, move each digit one place to the right.*

*To divide by 100, move each digit two places to the right.*

## Multiplication and Division

## Year 5

## Multiplying and Dividing by 10 and 100 (3).

**Vocabulary:**

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse Ones Tens Hundreds Tenths Hundredths

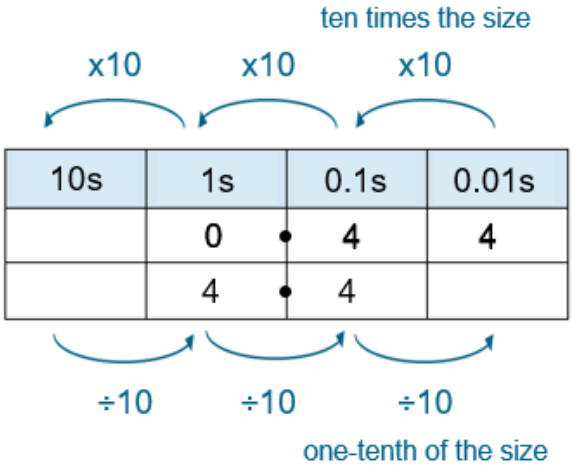
**0.27 × 10 = 2.7**

**2.7 ÷ 10 = 0.27**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1,000 | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 | 9,000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |

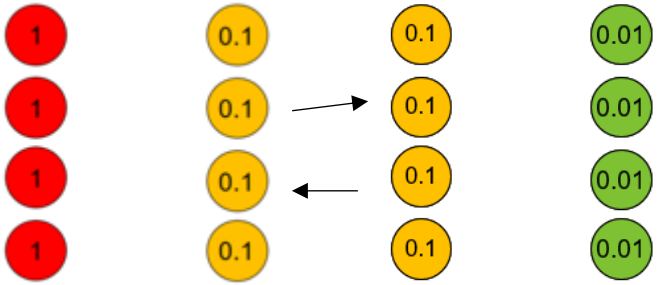
*0.27 is one-tenth the size of 2.7*

*2.7 divided by 10 is 0.27.*




**4.4 ÷ 10 = 0.44**

one-tenth of the size




**0.44 × 10 = 4.4**

ten times the size

*__ divided by 10/100 is equal to__.*

*__ is one-tenth/hundredth the size of __.*

*__ multiplied by 10/100 is equal to__.*

*__ is 10/100 times the size of __.*

**We can multiply and divide numbers with digits greater than 0 by 10 or 100.**

**Generalisation**

*To multiply by 10, move each digit one place to the left.*

*To multiply by 100, move each digit two places to the left.*

*To divide by 10, move each digit one place to the right.*

# Multiplication and Division

## Year 5

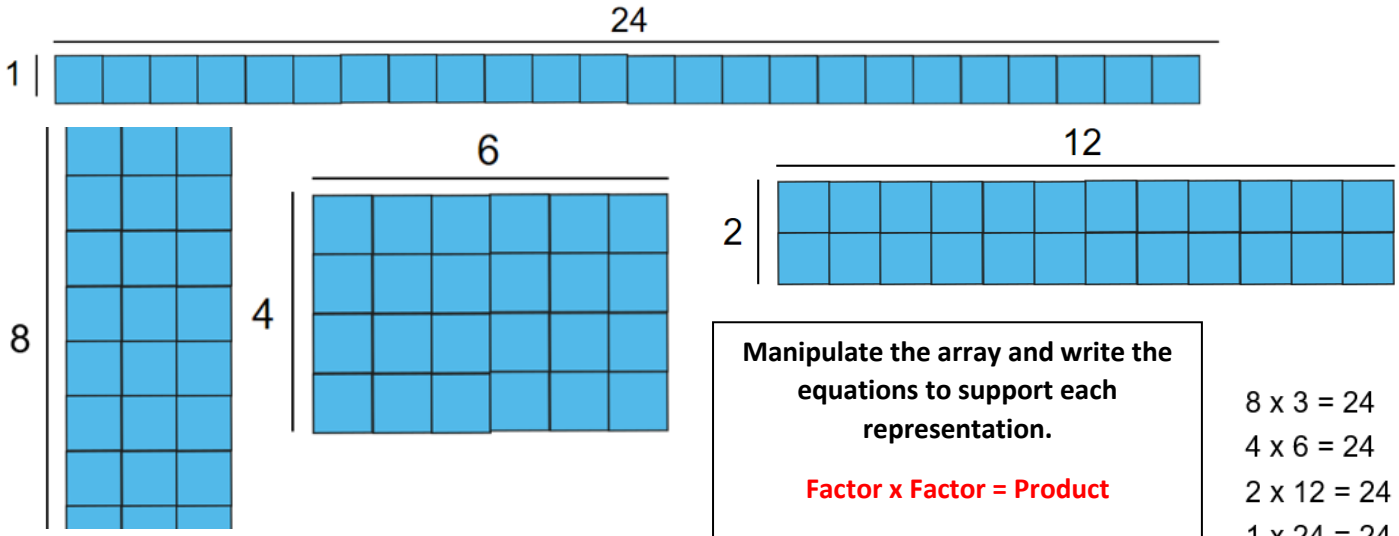
## Find Factors and Multiples (1)

**Vocabulary:**

Factor  Multiple  Composite  Square  Prime  Common Factor  Prime Factor

Factor Bug  Array  Positive Integer  Working Systematically

Factor x Factor = Product  Dividend ÷ Divisor = Quotient

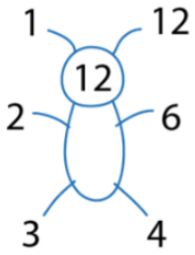



**Manipulate the array and write the equations to support each representation.**

**Factor x Factor = Product**

**Use factor bugs to record pairs of factors.**

8 x 3 = 24
4 x 6 = 24
2 x 12 = 24
1 x 24 = 24




**There are ___ tiles. There are ___ rows and ___ columns. So ___ and ___ are factors of ___.**

**Generalise: Numbers that have more than two factors are composite numbers.**

Equations can also be recorded with the product on the left of the equals symbol to develop flexibility.

**24 = 1 x 24**
**24 = 2 x 12**
**24 = 3 x 8**
**24 = 4 x 6**





**Generalisations**

***When one is a factor, the product is equal to the other factor.***

***All positive integers have a factor of 1.***

***Every positive integer is a factor of itself.***

***The smallest factor of a positive integer is always 1. The largest factor of a positive integer is always itself.***

# Multiplication and Division

## Year 5

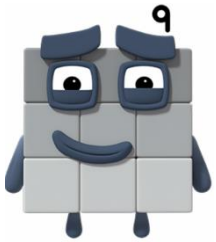
## Find Factors and Multiples

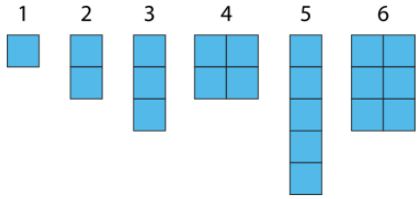
**Vocabulary:**

Factor  Multiple  Composite  Square  Prime  Common Factor  Prime Factor

Factor Bug  Array  Positive Integer  Working Systematically

Factor x Factor = Product  Dividend ÷ Divisor = Quotient




**Extend this to square numbers, and prime numbers recognising the number of factors.**

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 0 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

**Make connections with factors and times tables. Make connections with factors of factors**

***___ is a factor of ___ because it is in the ___ times table.***

***Nine is a factor of all of these numbers.***

***Three is a factor of nine which means it is also a factor of all of these numbers.***

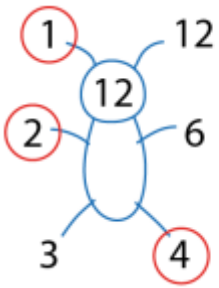
**Is 9 a factor of 54?**

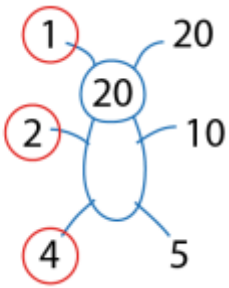
**54 ÷ 9 = 6**

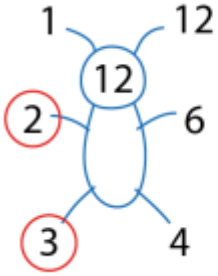
**9 and 6 are factors of 54.**

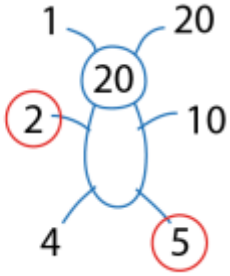
**Use factor bugs to find**

**common factors  and  prime factors.**




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

# Multiplication and Division

## Year 5

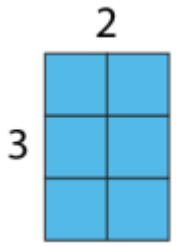
## Find Factors and Multiples

**Vocabulary:**

Factor  Multiple  Composite  Square  Prime  Common Factor  Prime Factor

Factor Bug  Array  Positive Integer  Working Systematically

Factor x Factor = Product  Dividend ÷ Divisor = Quotient




**Introduce Multiples**

**___ is a factor of ___ because ___ x ____ = ____.**

**___ is a multiple of ___ because ___ x ___ = ____**

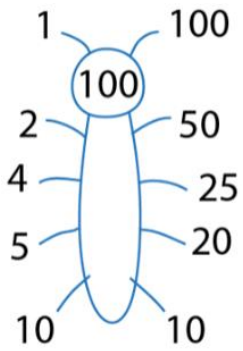
**___ is a factor of ___ because ___ ÷ ____ = ____.**

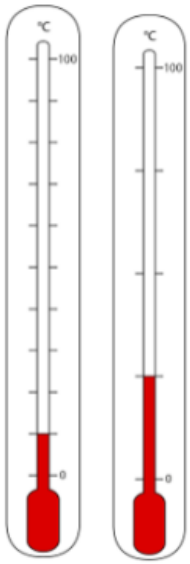
**___ is a multiple of ___ because ___ ÷ ___ = ____**

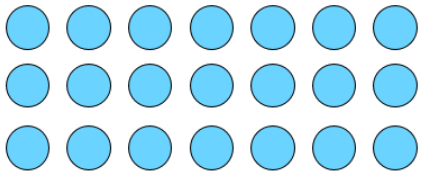
**Identify Common Multiples using a 100 square.**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

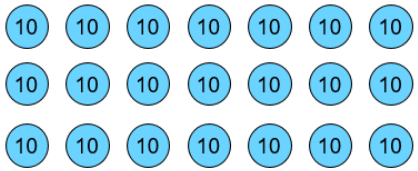
**Factors of 100 can be applied to real contexts**





7




**7 × 30 = 210**

**70 × 3 = 210**

**10 x 21 = 210**

**Make statements about factors and multiples whilst increasing the value of each counter in the array.**

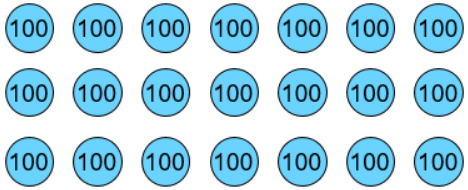
***___ represents the number of counters in each row.***

***___ represents the total value of the counters in each column.***

***____ represents the total value of the counters.***

***3, 7, 10, 21 and 70 are factors of 210.***

***210 is a multiple of 3, 7, 10, 21 and 70.***




**7 × 300 = 2,100**

**700 × 3 = 2,100**

**100 x 21 = 2,100**

# Multiplication and Division

## Year 5

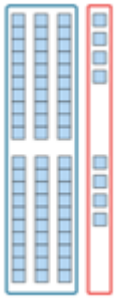
## Multiply using a Formal Written Method (1)

**Vocabulary:**

Ones Tens Hundreds Thousands Represents Partition Recombine Multiply Unitising Partial Product Aligned Calculation Expanded layout Compact layout Equation Regroup Algorithm

Factor x Factor = Product

NCETM guidance in Segment 2.14 includes expanded column multiplication. At Thorndown, we provide practice using informal strategies, where working is recorded using horizontal equations. Children become confident in multiplying 2-digit x 1 digit and 3-digit x 1-digit, before we move straight to compact columnar multiplication, initially representing each step of the calculation with dienes or place value counters. This enables children to work efficiently. Links are made with column addition started in Y3: aligning digits, starting from the left with the least significant digit, regrouping if there are ten or more ones.

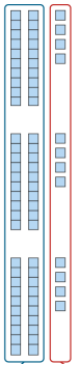



$34 = 30 + 4$

$34 \times 2 = 30 \times 2 + 4 \times 2$

$= 60 + 8$

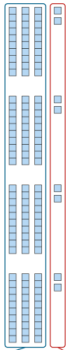
$= 68$



$24 \times 3 = 20 \times 3 + 4 \times 3$

$= 60 + 12$

$= \mathbf{72}$



$32 \times 4 = 30 \times 4 + 2 \times 4$

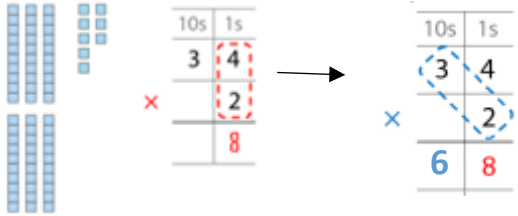
$= 120 + 8$

$= \mathbf{128}$

**Represent the multiplication using dienes.**
**Partition the number into tens and ones.**
**Multiply the tens and ones and then recombine.**
Children should be able to do this mentally. The dienes is a tool to represent the structure, not to do the maths.

**Move to the compact algorithm, alongside dienes.**
**Use the same unitising language as for columnar addition.**
**See Y3, p19 above.**

**34 x 2**

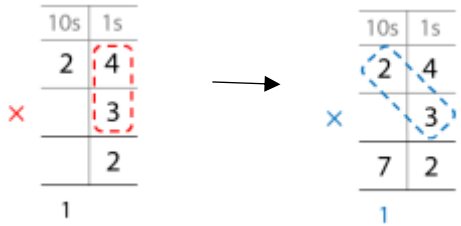



**4 ones x 2 = 8 ones**

**3 tens x 2 = 6 tens**

**Move to the compact algorithm, involving regrouping.**
**Use the same unitising language as for columnar addition.**
**See Y3, p20 above.**

**24 x 3**




**4 ones x 3 = 12 ones. We regroup 12 ones into 1 ten and 2 ones.**

**2 tens x 3 = 6 tens. We add the regrouped ten to make 7 tens.**

# Multiplication and Division

## Year 5

## Multiply using a Formal Written Method (2)

**Vocabulary:**

Ones Tens Hundreds Thousands Represents Partition Recombine Multiply Unitising Partial Product Aligned Calculation Expanded layout Compact layout Equation Regroup Algorithm

Factor x Factor = Product

**Extend to multiplication involving larger numbers, with and without regrouping in different columns. Initially represent using PV counters.**

367 × 4 = 1,468

| | 3 | 6 | 7 |
|---|---|---|---|
| × | | | 4 |
| 1 | 4 | 6 | 8 |
| | 2 | 2 | |

Multiplication algorithm – compact layout:

| | 2 | 6 |
|---|---|---|
| × | | 4 |
| 1 | 0 | 4 |
| | 2 | |

Multiplication algorithm – compact layout:

| | 3 | 8 |
|---|---|---|
| × | | 4 |
| 1 | 5 | 2 |
| | 3 | |

**If there are 10 or more ones, we must regroup ones into tens and ones.**
**If there are 10 or more tens, we must regroup into hundreds and tens.**
**If there are 10 or more hundreds, we must regroup into thousands and hundreds.**

Estimation – example 1:

24 × 3

- *'Twenty-four is between twenty and thirty.'*

20 × 3 = 60

30 × 3 = 90

- *'So, twenty-four times three must be between sixty and ninety.'*

Using inequalities and estimating – example practice:

- *'Fill in the missing numbers to complete this estimation.'*

48 × 6

48 is between 40 and 50.

40 × 6 = ☐

50 × 6 = ☐

So 48 × 6 must be between ___ and ___.

Calculations with missing digits:

*'Fill in the missing digits.'*

| | 5 | ☐ |
|---|---|---|
| × | | 3 |
| ☐ | ☐ | 9 |

| | 7 | 2 |
|---|---|---|
| × | | ☐ |
| 1 | 4 | ☐ |

Dòng nǎo jīn:

*'Fill in the missing digits.'*

**a**

| | 3 | ☐ |
|---|---|---|
| × | | 7 |
| 2 | 4 | 5 |

**b**

| | 3 | ☐ |
|---|---|---|
| × | | ☐ |
| 2 | 2 | 8 |

**Provide practice in using estimation skills to ensure children can reason about the reasonableness of their answers. This should become part of their normal practice.**

**Deepen understanding using empty boxes.**

# Multiplication and Division

## Year 5

## Divide using a Formal Written Method (1)

**Vocabulary:**

Partitive (sharing) Quotitive (grouping) Ones Tens Hundreds Thousands Represents Divide Unitising Dividend Divisor Quotient Partial Quotient Aligned Calculation Equation Exchange Algorithm 'Sharees' Divisible Remainder Short Division

| | | | | | |
|---|---|---|---|---|---|
| 84 | ÷ | 4 | = | 21 | $4\overline{)84}$ (2 1) |
| dividend | ÷ | divisor | = | quotient | divisor ⟌ dividend (quotient) |

**Use dienes to represent the division context where the dividend divides to give a whole number.**

84 sticks shared equally between 4 children. How many sticks each?

84 ÷ 4 = ☐

Step 1 – write the divisor and dividend:

| 10s | 1s |
|---|---|
| | |

$4\overline{)\ 8\quad 4}$

Step 2 – share the 10s:

| 10s | 1s |
|---|---|
| 2 | |

$4\overline{)\ 8\quad 4}$

8 tens ÷ 4 = 2 tens

8 tens divided by 4 is equal to 2 tens.

Step 3 – share the 1s:

| 10s | 1s |
|---|---|
| 2 | 1 |

$4\overline{)\ 8\quad 4}$

8 tens ÷ 4 = 2 tens

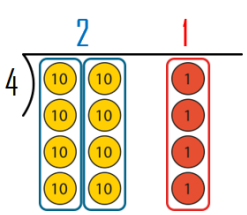
4 ones ÷ 4 = 1 one

**Add the partial quotients to find the quotient.**

**2 tens + 1 one = 21**

| 10s | 1s |
|---|---|
| 2 | 1 |

$4\overline{)\ 8\quad 4}$



8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

| | | | | | |
|---|---|---|---|---|---|
| 84 | ÷ | 4 | = | 21 | $4\overline{)84}$ (2 1) |
| dividend | ÷ | divisor | = | quotient | divisor ⟌ dividend (quotient) |

# Multiplication and Division

## Year 5

## Divide using a Formal Written Method (2)

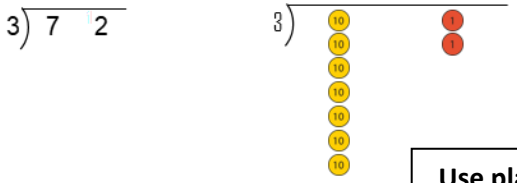
**Vocabulary:**

Partitive (sharing) Quotitive (grouping) Ones Tens Hundreds Thousands Represents Divide Unitising Dividend Divisor Quotient Partial Quotient Aligned Calculation Equation Exchange Algorithm Divisible Remainder Short Division

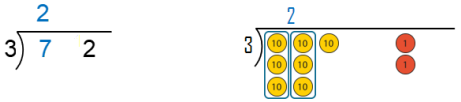
72 sticks shared equally between 3 children. How many sticks each?

72 ÷ 3 = ☐

Step 1 – write the divisor and the dividend:

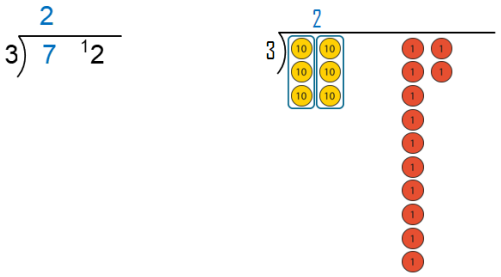


Step 2 – share the 10s:



7 tens ÷ 3 = 2 tens r 1 ten

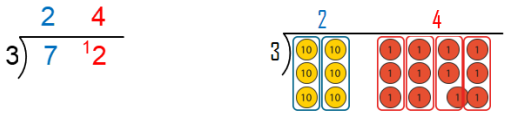
Step 3 – exchange:



7 tens ÷ 3 = 2 tens r 1 ten

72 ÷ 3 = 24

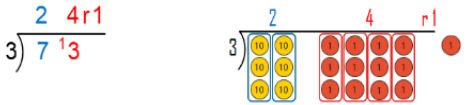
Step 4 – share the 1s:



7 tens ÷ 3 = 2 tens r 1 ten

12 ones ÷ 3 = 4 ones

73 ÷ 3 = 24 r 1



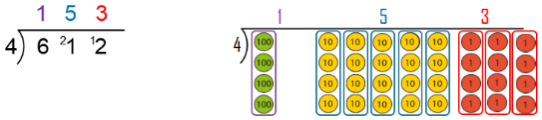
7 tens ÷ 3 = 2 tens r 1 ten

13 ones ÷ 3 = 4 ones r 1 one

**Use place value counters to represent a division context where the dividend divides to give a whole number though requires an exchange from the tens.**

**If dividing the tens gives a remainder of one or more ten, we must regroup and exchange the remaining tens for ones.**

612 ÷ 4 = 153



6 hundreds ÷ 4 = 1 hundred r 2 hundreds

2 hundreds = 20 tens

21 tens ÷ 4 = 5 tens r 1 ten

1 ten = 10 ones

12 ones ÷ 4 = 3 ones

**Apply the same representations though this time include a remainder.**

**Then extend to division of 3 digits by one digit and where there can be no hundreds cannot be shared.**

**If dividing the hundreds gives a remainder of one or more hundred, we must exchange the remaining hundreds for tens.**

## Addition, Subtraction, Multiplication and Division

## Year 6

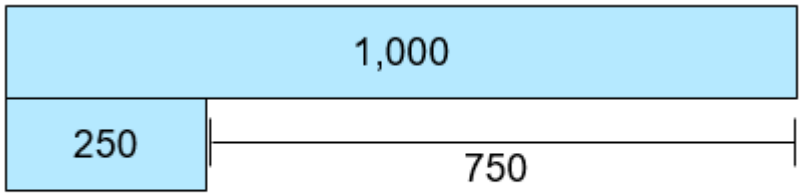
## Quantify additive and multiplicative relationships

**Vocabulary:**

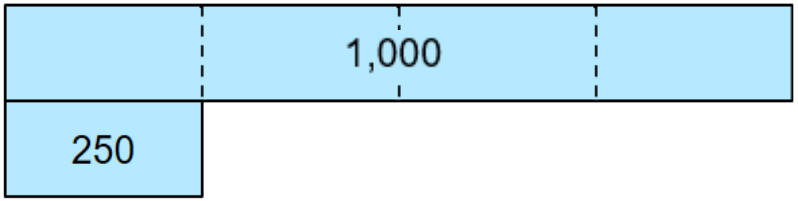
Additive Multiplicative Relationship Represents Compose Combine Total More than Less than Plus + Minus - Equal to = Addition Subtraction Divide ÷ Multiply x One-_____ of Equation Expression Bar Model Whole Part Difference Multiplier Unknown Sequence

Addend + Addend = Sum Factor x Factor = Product (Multiplicand x Multiplier = Product)

Minuend – Subtrahend = Difference Dividend ÷ Divisor = Quotient

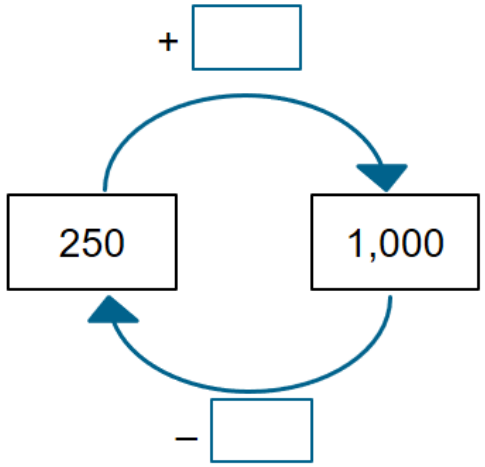



250 + 750 = 1,000 1,000 – 750 = 250



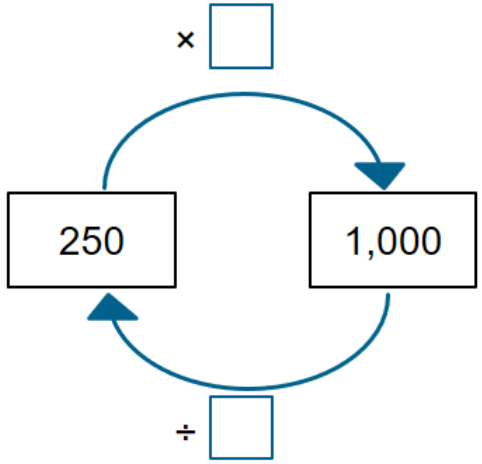

250 x 4 = 1,000 1000 ÷ 4 = 250

**The relationship between two numbers can be expressed both additively and multiplicatively.**




**1000 is ___ more than 250.**

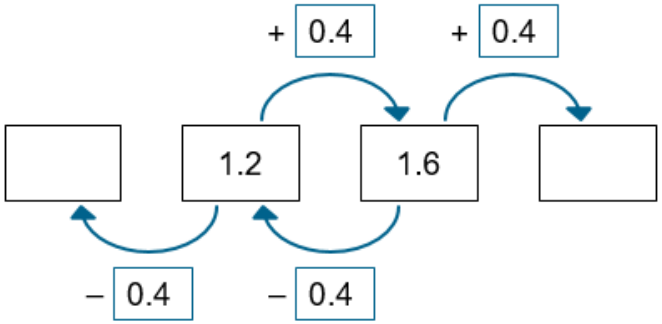
**250 is ____ less than 1000.**




**1000 is ___ times the size of 250.**

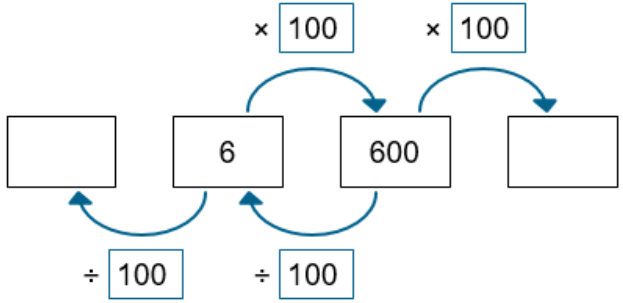
**250 is one-_____ of 1000.**

**To find one-quarter of a number, we divide by 4.**




**Finding the difference can help calculate the unknown terms in a sequence.**

**Finding the known multiplier can help calculate the unknown terms in a sequence.**



## Addition, Subtraction, Multiplication and Division

## Year 6

## Quantify additive and multiplicative relationships

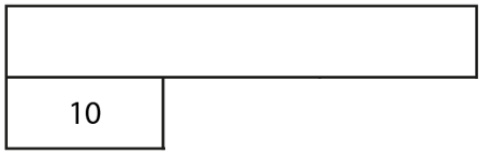
**Vocabulary:**

Additive Multiplicative Relationship Represents Compose Combine Total More than Less than Plus + Minus - Equal to = Addition Subtraction Divide ÷ Multiply x One-_____ of Equation Expression Bar Model Whole Part Difference Multiplier Unknown Sequence

Addend + Addend = Sum Factor x Factor = Product (Multiplicand x Multiplier = Product)

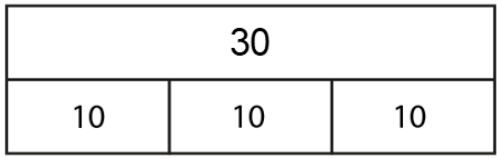
Minuend – Subtrahend = Difference Dividend ÷ Divisor = Quotient

$\frac{1}{3}$ of ?= 10




**Calculate the unknown whole by recognising the number of parts into which the whole has been divided.**

$\frac{1}{3}$ of ?= 10




$\frac{1}{3}$ of 30 = 10

## Addition and Subtraction

## Year 6

## Derive Related Calculations

**Vocabulary:**

Additive Multiplicative Relationship Represents Equation Unknown Re-arrange Inverse Place Value Properties Commutative Associative Distributive Compensation

Addend + Addend = Sum Factor x Factor = Product (Multiplicand x Multiplier = Product)

Minuend – Subtrahend = Difference Dividend ÷ Divisor = Quotient

| | | |
|---|---|---|
| $252 = 3 \times 84$ | $252 = 3 \times 84$ | $252 = 3 \times 84$ |
| $2{,}520 = 30 \times \square$ | $\square = 3 \times 85$ | $252 = 3 \times 60 + 3 \times \square$ |

| | | |
|---|---|---|
| $625 - 148 = 477$ | $625 - 148 = 477$ | $625 - 148 = 477$ |
| $6{,}250 - 1{,}480 = \square$ | $625 - 70 - \square = 477$ | $625 - 248 = \square$ |

| | | |
|---|---|---|
| $14.8 + 7.6 = 22.4$ | $14.8 + 7.6 = 22.4$ | $14.8 + 7.6 = 22.4$ |
| $1{,}480 + \square = 2{,}240$ | $\square - 7.6 = 14.8$ | $12.8 + \square = 22.4$ |

| | | |
|---|---|---|
| $4{,}800 \div 25 = 192$ | $4{,}800 \div 25 = 192$ | $4{,}800 \div 25 = 192$ |
| $25 \times 192 = \square$ | $4{,}800 \div 250 = \square$ | $4{,}800 \div 5 \div 5 = \square$ |

**Manipulate an equation to solve another. Pupils could:**

- **rearrange the terms;**
- **rewrite using inverse operations;**
- **apply place value;**
- **use the properties of division that correspond to the commutative, associative or distributive property of multiplication;**
- **use the compensation property.**

**Additive examples**

**Multiplicative examples**

# Addition and Subtraction

## Year 6

## Solve Problems involving Ratio Relationship

**Vocabulary:**
Additive Multiplicative Relationship Represents Equation Unknown Scale-factor Ratio Ratio Table ___ times the size one-___ the size of Vertical Horizontal
Factor x Factor = Product (Multiplicand x Multiplier = Product)
Dividend ÷ Divisor = Quotient


$1 \times 5 = 5$

$5 \div 5 = 1$

$5 \times \frac{1}{5} = 1$

$3 \times 5 = 15$

$15 \div 5 = 3$

$15 \times \frac{1}{5} = 3$

**Ratio table to compare sets of information.**

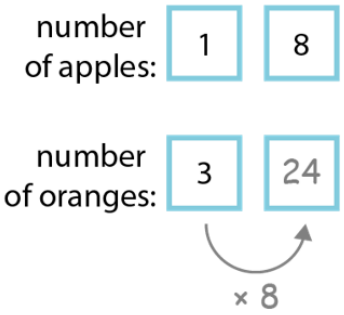
**For every ___, there are ___.**

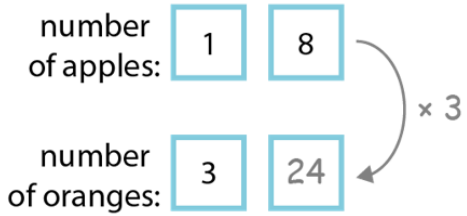
**For every 1 litre of petrol, you can drive 7 miles.**

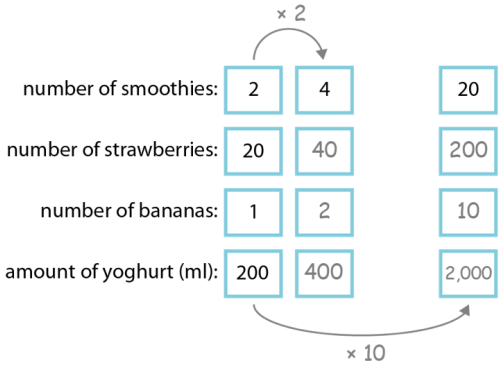
**For every 7 miles you will drive, you need 1 litre of petrol.**

**Extend sequences using knowledge of patterns based on ratio table.**

| Litres of petrol | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Miles driven | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |




**Explore vertical and horizontal relationship between numbers.**

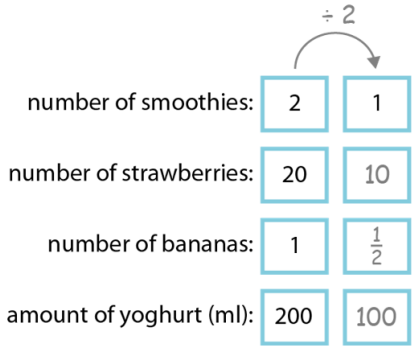
**For every ___, there are ___.**

**Identify the scale-factor in order to find unknown values.**

**___ is ___ times the size of ___.**

**Therefore, I must multiply/divide by ___.**

**___ is one-__ the size of ___.**



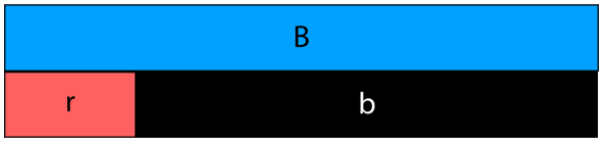
## Addition and Subtraction

## Year 6

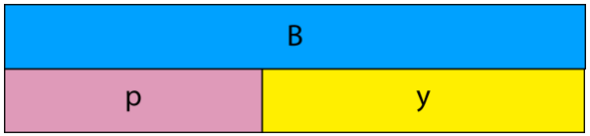
## Solve Problems with Two Unknowns

**Vocabulary:**

Additive Multiplicative Relationship Represents Equation Two Unknowns Scale-factor Ratio ___ times the size one-___ the size of Total Bar Model Structure




$B = r + b$




$B = p + y$

**Use Cuisenaire to find 2 bars of total length that are equal to another.**

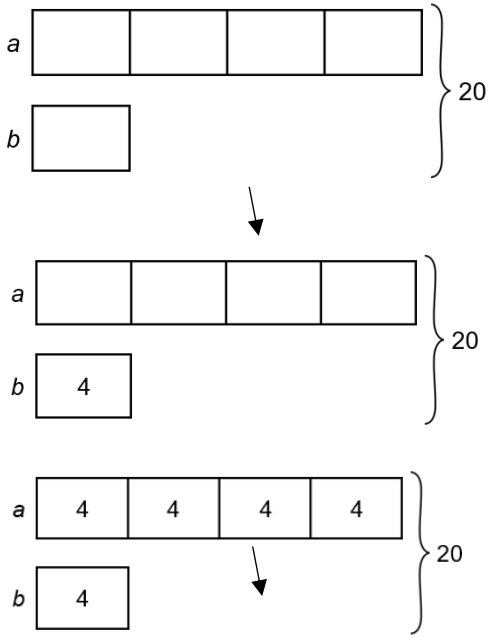
**There is more than one solution to the problem.**

**There can be infinite solutions to a problem.**

$5 \times \square = 10 \times \square$

**Solve multiplicative problems with two unknowns when the total is known.**

v



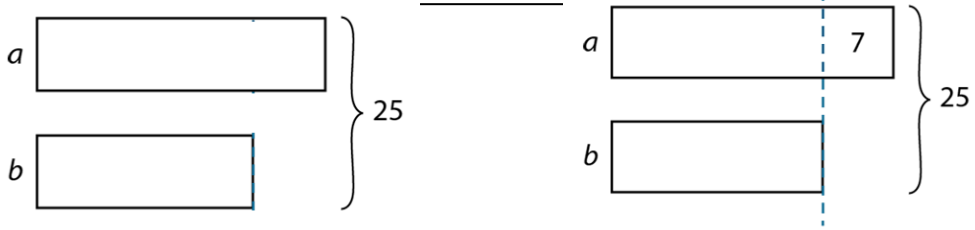

one part = 20 ÷ 5 = 4

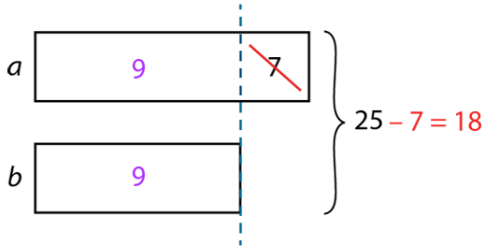
$b = 4$

$a = 4 \times 4 = 16$

*The two numbers are 16 and 4.*

**Solve additive problems with two unknowns when the total is known**




$b = 18 \div 2 = 9$

$a = 9 + 7 = 16$

*The two numbers are 9 and 16.*